LTAM Exam

Fall 2020

Solutions to Written Answer Questions

Question 1 Model Solution

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a)

Let * denote the extra mortality.

(i) $\text{EPV} = 200,000 \times {}_{20}E_{45}^{*}$

$$ {}_{20}E_{45}^{*} = {}_{20}p_{45}^{*}\ e^{-20\delta} $$

$$ {}_{t}p_{x}^{*} = e^{-\int_0^t \left(\mu_{x+s}^{SULT} + 0.02\right) ds} = e^{-0.02t}\ {}_{t}p_{x}^{SULT} $$

$$ \Rightarrow {}_{20}E_{45}^{*} = {}_{20}p_{45}^{SULT}\ e^{-20(\delta+0.02)} = \frac{94,579.7}{99,033.9} \times \mathrm{e}^{-20(0.065)} = 0.2602743 $$

$$ EPV = 200{,}000 \times 0.2602743 = 52{,}054.9 $$

Comment: This part was done correctly by almost all candidates.

(ii) $\text{EPV} = 4000 \times 12 \times \ddot{a}_{45}^{*(12)} = 4000 \times 12 \times \left(\ddot{a}_{45}^{*} - \frac{11}{24}\right)$

$$ = 4000 \times 12 \times \left(14.4893 - \frac{11}{24}\right) = 48,000 \times 14.030967 = 673,486 $$

Comment: This part was also done correctly by almost all candidates.

(iii) $\text{EPV} = 100,000\bar{A}^{*\,1}_{45:\overline{20}|} = 100,000\left(1-\delta\bar{a}^{*}_{45:\overline{20}|} - {}_{20}E^{*}_{45}\right)$

$$\bar{a}^{*}_{45:\overline{20}|} = \left(\ddot{a}^{*}_{45:\overline{20}|} - \frac{1}{2}\left(1 - {}_{20}E^{*}_{45}\right)\right) = \left(11.4411 - \frac{1}{2}\left(1-0.2602743\right)\right) = 11.0712$$

$\Rightarrow$ $\text{EPV} = 100,000\left(1 - 0.045 \times 11.0712 - 0.2602743\right) = 24,152$

Comment: Candidates did not do as well this part. Common mistakes were:

- *Forgetting to include the 12 in the calculation*
- *Assuming that death were uniformly distributed between integral ages.*
- *Assuming the death benefit was paid at the end of the year of death instead of at the moment of death*
- *Treating the benefit as an endowment insurance instead of a term insurance*

b)

$${}_{20}V = 4000 \times 12 \times \ddot{a}_{65}^{*(12)} = 48{,}000 \times \left(\ddot{a}_{65}^{*} - \frac{11}{24} \right)$$

$$\ddot{a}_{45}^{*} = \ddot{a}_{45:\overline{20}|}^{*} + {}_{20}E_{45}^{*} \times \ddot{a}_{65}^{*}$$

$$\Rightarrow \ddot{a}_{65}^{*} = \frac{14.4893 - 11.4411}{0.2602743} = 11.7115$$

$$\Rightarrow \ddot{a}_{65}^{*(12)} = 11.7115 - \frac{11}{24} = 11.2532$$

$$\Rightarrow {}_{20}V = 48{,}000 \times 11.2532 = 540{,}152$$

Alternative Solution:

$${}_{20}V = 4000 \times 12 \times \ddot{a}_{65}^{*(12)}$$

$$\ddot{a}_{45:\overline{20}|}^{*(12)} = \ddot{a}_{45:\overline{20}|}^{*} - \frac{11}{24}(1 - {}_{20}E_{45}^{*}) = 11.4411 - \frac{11}{24}(1 - 0.2602743) = 11.102059$$

$$\ddot{a}_{45:\overline{20}|}^{*(12)} = \ddot{a}_{45}^{*(12)} - {}_{20}E_{45}^{*} \times \ddot{a}_{65}^{*(12)} = 11.102059 = 14.030967 - (0.2602743)\ddot{a}_{65}^{*(12)}$$

$$\ddot{a}_{65}^{*(12)} = \frac{14.030967 - 11.102059}{0.2602743} = 11.253159$$

$$\Rightarrow {}_{20}V = 48{,}000 \times 11.2532 = 540{,}152$$

Comment: Candidates did not do well this part. Common mistakes were:

- *Including the survival benefit in the reserve*
- *Forgetting to include the 12 in the calculation*
- *Treating the annuity as being paid annually instead of monthly.*

c)

(i) The EPV would not change.

Survival benefits depend on $\mu^*_{x+t} + \delta$. In part (a), we had

$\mu^*_{x+t} + \delta = \mu^{SULT}_{x+t} + 0.02 + 0.045 = \mu^{SULT}_{x+t} + 0.065.$

Now we have

$\mu^*_{x+t} + \delta = \mu^{SULT}_{x+t} + 0.01 + 0.055 = \mu^{SULT}_{x+t} + 0.065.$

The result is that life annuity benefits will have the same value under the new assumptions as they did under the old assumptions.

(ii) The EPV would decrease.

Argument 1: In this case, the lower mortality reduces the EPV of the death benefit, and the higher interest also makes the death benefit cheaper, so the overall effect will be a reduction in EPV.

Or

Argument 2: Since $\bar{A}_x = 1 - \delta\bar{a}_x$ and $\bar{a}_x$ does not change (see c.i.) then when δ increases $\bar{A}_x$ will decrease. Similarly, the same argument would hold for a term insurance.

Or

Argument 3: $\bar{A}^{1}_{x:\overline{20}|} = \int_0^{20} v^t \cdot {}_t p_x \cdot \mu_{x+t} \cdot dt$. The changes mortality and interest to result in $v^t \cdot {}_t p_x$ having the same value under either set of assumptions. However, since μ_{x+t} is smaller, the present value of the death benefit will be smaller.

Comment: Candidates did reasonably well this part. Most students correctly deduced that the life annuity benefits did not change in value. Some students then incorrectly decided that the death benefit would also not change in value. Some students did not sufficiently support their conclusions with sufficient justification.

Question 2 Model Solution

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a)

(i) $_t p_x = e^{-\int_0^t \mu_{x+s}\, ds}$

$$\int_0^t \mu_{x+s}\, ds = \int_0^t \alpha e^{\lambda(x+s)}\, ds = \left[\frac{\alpha}{\lambda} e^{\lambda(x+s)}\right]_0^t$$

$$= \frac{\alpha}{\lambda}\left(e^{\lambda(x+t)} - e^{\lambda x}\right)$$

$$= \frac{\alpha}{\lambda} e^{\lambda x}\left(e^{\lambda t} - 1\right)$$

$$\Rightarrow {}_t p_x = e^{-\left\{\frac{\alpha}{\lambda} e^{\lambda x}(e^{\lambda t} - 1)\right\}} \quad \text{as required}$$

(ii) Now $\mu_x = \alpha$,

$$\Rightarrow \qquad \int_0^t \mu_{x+s}\, ds = \alpha t \Rightarrow {}_t p_x = e^{-\alpha t}$$

Comments:

- *Most candidates did very well on part (i)*
- *Alternative approaches include recognizing that this is Makeham with A=0, B=α, and c = e$^{\lambda}$ or Gompertz with B=α and c = e$^{\lambda}$, or differentiating the given function to derive the given force of mortality.*
- *Part (ii) was less well done. Many candidates plugged λ = 0 into the given expression and failed to deal with the resulting 0/0, e.g. by using L'hopital's rule*

b)

(i) First: ${}_0p_x = 1$

Second: $\lim_{t\to\infty} {}_tp_x = 0$

Third: ${}_tp_x$ is a decreasing (or non-increasing) function of *t*.

(ii) First: ${}_0p_x = e^{-\frac{\alpha}{\lambda}e^{\lambda x}(e^0-1)} = e^0 = 1$

Second: $\lim_{t\to\infty} {}_tp_x = \lim_{t\to\infty} e^{-\left\{\frac{\alpha}{\lambda}e^{\lambda x}(e^{\lambda t}-1)\right\}} = 0$

Third:

$$\frac{d}{dt}{}_tp_x = \frac{d}{dt}\left(e^{-\left\{\frac{\alpha}{\lambda}e^{\lambda x}(e^{\lambda t}-1)\right\}}\right)$$

$$= \left(\lambda e^{\lambda t}\right)\times\left(-\frac{\alpha}{\lambda}e^{\lambda x}\right)\times\left(e^{-\left\{\frac{\alpha}{\lambda}e^{\lambda x}(e^{\lambda t}-1)\right\}}\right) \text{ (Chain Rule)}$$

$$= -\left(\alpha e^{\lambda(x+t)}\right)\times\left(e^{-\left\{\frac{\alpha}{\lambda}e^{\lambda x}(e^{\lambda t}-1)\right\}}\right)$$

$$\alpha,\lambda > 0 \Rightarrow \alpha e^{\lambda(x+t)} > 0; \quad e^{-\left\{\frac{\alpha}{\lambda}e^{\lambda x}(e^{\lambda t}-1)\right\}} > 0$$

$$\Rightarrow \frac{d}{dt}{}_tp_x < 0$$

So the three conditions are satisfied

(iii) First: ${}_0p_x = e^{-0} = 1$

Second: $\lim_{t\to\infty} {}_tp_x = \lim_{t\to\infty} e^{-\alpha t} = 0$

Third: $\frac{d}{dt}{}_tp_x = -\alpha e^{-\alpha t} < 0$

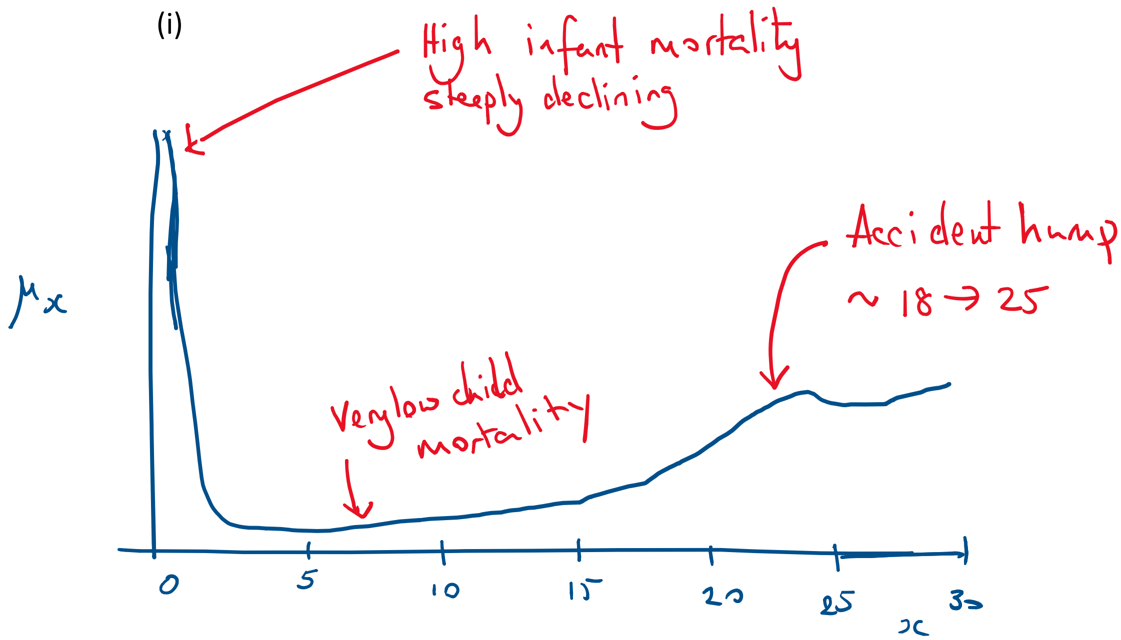
So the three conditions are satisfied when λ=0.

Comments:

- *Most candidates remembered the three conditions correctly.*
- *Demonstrating the first two conditions was also fairly well done.*
- *Alternative approaches for showing the function is decreasing include showing that* $\log {}_tp_x$ *is decreasing, showing that* ${}_sp_x < {}_tp_x$ *for all t < s, or showing that* ${}_sp_x/{}_tp_x < 1$ *for all t < s.*
- *Some candidates only proved the function was decreasing for specific values of t, x, λ, or α – this earned partial credit*

c)

(i)



(ii) The model assumes exponentially increasing force of mortality. For this age group, the force of mortality is decreasing at very young ages, and flattens or decreases after the accident hump, during the late 20's.

Comments:

- *The sketch does not have to be exact or to scale, but the three key features should be included.*
- *This part was the most poorly done. Many candidates misunderstood the question and graphed the function they were given for the force of mortality, instead of the mortality for typical lives.*

Question 3 Model Solution

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a)

T	*S(t)*	*S(t)*
$0 \le t < 1.5$	1.0	1.0
$1.5 \le t < 3.0$	29/30	0.96667
$3.0 \le t < 4.2$	$29/30 \times 27/28$	0.93214
$4.2 \le t < 8.9$	$29/30 \times 27/28$ $\times 25/27$	0.863095

Comment: Most candidates got this correct.

b)

$$EPV = 100,000\; S(5)\; v_{6\%}^{5} = (100,000)(0.863095)(1.06)^{-5} = 64,495.5$$

Comment: Most candidates got full marks in this question. Several candidates calculated the EPV for an endowment insurance instead of the pure endowment insurance asked.

c)

The reserve is

$$_3p_{77} \times 100,000 \times v_{6\%}^{3} = \left(\frac{S(5)}{S(2)}\right)(100,000)(1.05)^{-3} = 74,966$$

Or, recursively,

$$\left({}_0V + NSP\right)(1+i) = q_{75} \cdot 0 + p_{75} \cdot {}_1V$$

$$_1V = 64,495.5 \cdot \frac{1.06}{1} = 68,365.2$$

$$\left({}_1V + 0\right)(1+i) = q_{76} \cdot 0 + p_{76} \cdot {}_2V$$

$$_2V = 68,365.2 \cdot \frac{1.06}{0.96667} = 74,966$$

Comment: Candidates may use either approach. Common mistakes included failure to obtain the correct value for 3p77 or S(2).

d)

$$V[S(t)] \approx \left(S(t)\right)^2 \sum_{i:t_i<t} \frac{s_i}{r_i(r_i - s_i)}$$

$$V[S(5)] \approx \left(S(t)\right)^2 \left(\frac{1}{30 \times 29} + \frac{1}{28 \times 27} + \frac{2}{27 \times 25}\right)$$

$$\approx (0.863095)^2 \times 0.005435 \approx 0.004049$$

$$\Rightarrow SD[S(5)] \approx 0.06363$$

Comments: This question requires candidates to use Greenwood's variance formula to find the standard deviation for the Kaplan-Meier estimator of S(5). Most candidates did it well, but a few candidates even didn't know that the formula was provided in the tables with the exam.

e)

(i) The 95% CI is $0.86310 \pm 1.96 \times 0.06363$

$= (0.73838\ ;\ 0.98781)$

(ii) The 95% CI is

$\left(0.73838 \times 100,000 \times v^5 - 65,500\ ;\ 0.98781 \times 100,000 \times v^5 - 65,500\right)$

$= (-10,324\ ;\ 8315)$

Comments: Candidates did fairly well on this question. A number of candidates did not answer the question asked and instead provided a log-linear confidence interval for S(5) in (e) (i). For (e) (ii), some candidates considered the variance of the loss, ignoring (e) (i), even when the question specifically asked candidates to construct the CI using the result in (e) (i).

Question 4 Model Solution

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a)

The probability is:

$$\Pr[1 \text{ alive}] = 2 \times {}_{20}p_{50}(1 - {}_{20}p_{50})$$

$${}_{20}p_{50} = \frac{l_{70}}{l_{50}} = \frac{91{,}082.4}{98{,}576.4} = 0.92398$$

$$\Rightarrow (2)(0.92398)(1-0.92398) = 0.1405$$

Comment: This part was done correctly by almost all candidates.

b)

$$\ddot{a}^{(2)}_{\overline{50:50}:\overline{20}|} = 2\ddot{a}^{(2)}_{50:\overline{20}|} - \ddot{a}^{(2)}_{50:50:\overline{20}|}$$

$$\ddot{a}^{(2)}_{50:\overline{20}|} = \ddot{a}_{50:\overline{20}|} - \frac{1}{4}\left(1 - {}_{20}E_{50}\right) = 12.8428 - 0.25(1-0.34824) = 12.6799$$

$$\ddot{a}^{(2)}_{50:50:\overline{20}|} = \ddot{a}^{(2)}_{50:50} - {}_{20}E_{50} \times \ddot{a}^{(2)}_{70:70} = \left(\ddot{a}_{50:50} - \frac{1}{4}\right) - \left({}_{20}p_{50}\right)^2(1.05)^{-20}\left(\ddot{a}_{70:70} - \frac{1}{4}\right)$$

$$(15.8195-0.25) - (0.92398)^2(1.05)^{-20}(9.9774-0.25) = 12.4396$$

$$\Rightarrow \ddot{a}^{(2)}_{\overline{50:50}:\overline{20}|} = (2)(12.6799) - 12.4396 = 12.9202$$

Comment: Many students attempted this part, and most of them received partial credit for correct intermediate steps. The most common errors were:

- *Incorrect application of Woolhouse's formula or using a joint endowment instead of a last survivor when calculating the adjusted last survivor term annuity*
- *Incorrect Woolhouse coefficients (11/24 or ½ instead of 1/4)*

c)

EPV Benefits:

$$100,000\, A_{\overline{50:50}} = 100,000(2A_{50} - A_{\overline{50:50}}) = 100,000(2\times 0.18931 - 0.24669) = 13,193$$

EPV Prems $-$ Exp:

$$G\left\{\left(\ddot{a}^{(2)}_{\overline{50:50}:\overline{20|}} + 0.1405 v^{20}\ddot{a}_{70} + {}_{20}p_{50:50} v^{20}\ddot{a}_{\overline{70:70}}\right)\times 0.9 - 0.7\right\}$$

$$\ddot{a}_{\overline{70:70}} = 2\times\ddot{a}_{70} - \ddot{a}_{70:70} = (2)(12.0083) - 9.9774 = 14.0392$$

$$\Rightarrow \text{ EPV Prems } - \text{Exp} = G\left\{\left(12.9202 + 0.1405\times v^{20}\times 12.0083 + (0.92398)^2(v^{20})(14.0392)\right)0.9 - 0.7\right\}$$

$$= G\left\{0.9\times 18.0734 - 0.7\right\} = 15.5661G$$

$$\Rightarrow G = \frac{13,193}{15.5661} = 847.55$$

Comments: Not many students received full credit, but most of those that attempted received the majority of points for correctly setting up the EPV formulas. Common mistakes include:

- *Incorrect expense payment (60% vs. 70%)*
- *Missing the annuity portion due when exactly 1 person is alive*
- *Multiplying the first annuity term by 2 (incorrect treatment of semi-annual premium payment)*

d)

(i) Let ${}_{20}V^{(2)}$ denote the reserve conditional on both surviving:

$${}_{20}V^{(2)} = 100000A_{\overline{70:70}} - (0.9)(847.55)\ddot{a}_{\overline{70:70}}$$

$$A_{\overline{70:70}} = 1 - d\ddot{a}_{\overline{70:70}} = 1 - \left(\frac{0.05}{1.05}\right)(14.0392) = 0.33147$$

$$\text{or } (2)A_{70} - A_{70:70} = (2)(0.42818) - 0.52488 = 0.33148$$

$$\Rightarrow {}_{20}V^{(2)} = 22,437.6$$

(ii) Let ${}_{20}V^{(1)}$ denote the reserve conditional on exactly one surviving:

$${}_{20}V^{(1)} = 100,000A_{70} - (0.9)(847.55)\ddot{a}_{70} = (100,000)(0.42818) - (0.9)(847.55)(12.0083) = 33,658$$

Comments: Students who attempted this part generally received the majority of the credit. The most common mistake was neglecting the 90% coefficient on the annuity portion of the calculation.

e)

$$\left({}_{19.5}V^{(2)} + 0.9\times G/2\right)(1.05)^{0.5} = \underbrace{\left({}_{0.5}q_{69.5}\right)^2 \times 100{,}000}_{\text{Both Insureds Die}} + \underbrace{\left({}_{0.5}p_{69.5}\right)^2 \; {}_{20}V^{(2)}}_{\text{Both Insureds Live}} + \underbrace{2\left({}_{0.5}q_{69.5}\;\;{}_{0.5}p_{69.5}\right){}_{20}V^{(1)}}_{\substack{\text{One Insured Lives and One Dies} \\ \text{The (2) in front is because either could die.}}}$$

$${}_{0.5}p_{69.5} = \left(p_{69}\right)^{0.5} = (1-0.009294)^{0.5} = 0.99534$$

$$\Rightarrow {}_{19.5}V^{(2)} = \left[\frac{(1-0.99534)^2(100{,}000) + (0.99534)^2(22{,}437.6) + 2(0.99534)(1-0.99534)(33{,}658)}{(1.05)^{0.5}}\right] - (0.9)(847.55/2)$$

$$(1.05)^{-0.5}(2.17 + 22{,}228.97 + 312.23) - 0.9\times 847.55/2 = 21{,}619$$

Comments: Not many students attempted this part, and very few received full credit. When they attempted to solve for the survival probability, students generally performed the calculation correctly, the majority of mistakes came from the recursion equation:

- *Missing the premium payment or missing the coefficients*
- *Using the wrong interest rate or an annual time-step*
- *Not accounting for all future outcomes for both lives or accounting for the outcomes incorrectly (e.g. only calculating the roll-forward amount for the case that both or neither survive)*

Question 5 Model Solution

(a)

(i)

$L_0^G = 100{,}000v^{K_{40}+1} + 50\ddot{a}_{\overline{K_{40}+1}|} + 950 + 0.4G - 0.9G\ \ddot{a}_{\overline{K_{40}+1}|}$

OR $L_0^G = 100{,}000v^{K_{40}+1} + (50 - 0.9G)\ddot{a}_{\overline{K_{40}+1}|} + 950 + 0.4G$

OR $L_0^G = 100{,}000v^{K_{40}+1} + 50\ddot{a}_{\overline{K_{40}+1}|} + 950 - G\big(0.9\ \ddot{a}_{\overline{K_{40}+1}|} - 0.4\big)$

(ii)

$$E[L_0^G] = 0 = 100{,}000A_{40} + 50\ddot{a}_{40} + 950 - G(0.9\ddot{a}_{40} - 0.4)$$
$$\Rightarrow G = \frac{100{,}000A_{40} + 50\ddot{a}_{40} + 950}{(0.9\ddot{a}_{40} - 0.4)}$$

where:

EPV(Benefits): $100{,}000A_{40} = 12{,}106$

EPV(Expenses): $50\ddot{a}_{40} + 950 = 1{,}872.89$

EPV(Prem – Exp): G$(0.9\ddot{a}_{40} - 0.4) = 16.21202G$

$\Rightarrow G = 862.25$

Comments: Almost all candidates earned full credit on this part.

(b)

Median of T_{40} is τ such that

$${}_{\tau}p_{40} = 0.5 = \frac{l_{40+\tau}}{l_{40}} = \frac{l_{40+\tau}}{99{,}338.3} \Rightarrow l_{40+\tau} = 49{,}669.15$$

$\Rightarrow 48 < \tau < 49 \Rightarrow \lfloor\tau\rfloor = 48\ so\ K_{40} = 48$

$L_0^G|[K_{40} = 48] = 100{,}000v^{49} + 50\ddot{a}_{\overline{49}|} + 950 - (G)(0.9\ddot{a}_{\overline{49}|} - 0.4)$

where:

$G = 862.25$

$v^{49} = 0.091563913$

$\ddot{a}_{\overline{49}|} = 19.0771578$

$\Rightarrow L_0^G|[K_{40} = 48] = -3399$

Comments: Candidates who answer this question did very well but it was omitted my many candidates.

(c) Any of these explanations would be acceptable.

i. Under the equivalence principle, each of the individual policies has mean loss-at-issue equal to zero, so the expected value of the aggregate loss-at-issue is also zero.

ii. Also, the central limit theorem tells us that the distribution of the sum of i.i.d. random variables converges to a Normal Distribution as the sample size increases.

iii. Since the normal distribution is symmetric about its mean, the probability that the aggregate loss-at-issue on the portfolio is less than 0 is approximately 0.5.

Comments: This question was not answered correctly or was omitted by many candidates.

(d)

Let $S = 100{,}000$ and $K = K_{40}$

For the j^{th} policy we have:

$$L_j = Sv^{K+1} - (0.9 \times 900 - 50)\ddot{a}_{\overline{K+1}|} + 950 + 0.4 \times 900$$
$$L_j = Sv^{K+1} - (760)\ddot{a}_{\overline{K+1}|} + 1310$$

$$L_j = \left(S + \tfrac{760}{d}\right)v^{K+1} - \tfrac{760}{d} + 1310 = \left(S + \tfrac{760}{d}\right)v^{K+1} - 14{,}650 = 115{,}960v^{K+1} - 14{,}650$$

$$\Rightarrow E[L_j] = 115{,}960A_{40} - 14{,}650 = -611.88$$

$$\Rightarrow V[L_j] = 115{,}960^2({}^2A_{40} - A_{40}^2) = 115{,}960^2(0.02347 - 0.12106^2) = 118{,}525{,}810.2 = 10{,}886.96^2$$

Let $L = \sum_{j=1}^{100} L_j$ so that $L \sim N(100E[L_j], 100V[L_j]) \Rightarrow L \sim N(-61{,}188\,, 108{,}869.6^2)$

$$\Pr[L > 0] \approx 1 - \Phi\left(\frac{0 - (-61{,}188)}{108{,}869.6}\right) = 1 - \Phi(0.562) = 1 - \Phi(0.56) = \mathbf{0.2877}$$

Comment: The candidates that answered this question got most of the points but it was omitted by quite a few students.

(e)

There will be no change in the FPT reserve.

The FPT method is a net premium method, and therefore independent of the gross premium.

Comments: Very few students earned full credit on this part. Some students correctly stated that there was no change in the reserve but did not correctly justify the answer. Most students just omitted this part.

Question 6 Model Solution

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a)

$$AL = 20 \times 8 \times 45{,}000 \times 0.02 \times {}_{30}E_{35} \times \ddot{a}_{65}^{(12)}$$
$$+5 \times 25 \times 62{,}000 \times 0.02 \times \ {}_{5}E_{60} \times \ddot{a}_{65}^{(12)}$$
$$+32{,}000 \times \ddot{a}_{70}^{(12)}$$
$$\ddot{a}_{65}^{(12)} = \ddot{a}_{65} - \frac{11}{24} = 13.0915; \quad \ddot{a}_{70}^{(12)} = 11.5500; \quad {}_{30}E_{35} = 0.21981$$
$$\Rightarrow AL = 20 \text{ x } 20{,}719 + 5 \text{ x } 311{,}223 + 369{,}599$$
$$= 414{,}380 + 1{,}556{,}115 + 369{,}599 = 2{,}340{,}094$$

Comments: Candidates who seriously attempted this question did well on this part. Many candidates omitted this part.

b)

NC for age 35 group:

$$414{,}380 \times \left(1.025 \times \frac{9}{8} - 1\right) = 63{,}452$$

NC for age 60 group:

$$1{,}556{,}111 \times \left(1.025 \times \frac{26}{25} - 1\right) = 102{,}704$$

Alternate Solutions for NC using expected PV

${}_{29}E_{36} = 0.23089$, ${}_{4}E_{61} = 0.80796$

NC for age 35 group:

$NC_{35} = PV(AL_{36}) - AL_{35}; AL_{35}\ comes\ from\ part\ A\ and\ is\ 414{,}380$

$AL_{36} = 20 \text{ x } 9 \text{ x } (45{,}000 \text{ x } 1.025) \text{ x } {}_{29}E_{36} \text{ x } \ddot{a}_{65}^{(12)} = 501{,}920.2$

$PV(AL_{36}) = v \text{ x } p_{35} \text{ x } AL_{36} = 0.952381 \text{ x } 0.99961 \text{ x } 501{,}920.2 = 477{,}832.83$

$NC_{35} = PV(AL_{36}) - AL_{35} = 477{,}833 - 414{,}380 = 63{,}453$

NC for age 60 group:

$NC_{60} = PV(AL_{61}) - AL_{60}; AL_{60}\ comes\ from\ part\ A\ and\ is\ 1{,}556{,}113$

$AL_{61} = 5 \text{ x } 26 \text{ x } (62{,}000 \text{ x } 1.025) \text{ x } \ {}_{4}E_{61} \text{ x } \ddot{a}_{65}^{(12)} = 1{,}747{,}694.4$

$PV(AL_{61}) = v \text{ x } p_{60} \text{ x } AL_{61} = 0.952381 \text{ x } 0.99660 \text{ x } 1{,}747{,}694.4 = 1{,}658{,}812$

$NC_{60} = PV(AL_{61}) - AL_{60} = 1{,}658{,}812 - 1{,}556{,}113 = 102{,}699$

Total Salary

$$45{,}000 \times 20 \times 1.025 + 62{,}000 \times 5 \times 1.025 = 922{,}500 + 317{,}750 = 1{,}240{,}250$$

Normal Contribution Rate

$$\frac{63{,}452 + 102{,}704}{1{,}240{,}250} = 13.397\%$$

Comment: This part was not very well done. The candidates who omitted part a generally omitted this part. Additionally, for those that did answer, other mistakes led to less than full credit. Some students also had trouble expressing the normal cost as a percentage of total payroll.

c) 4 grading points

Using the numbers calculated in part (b), we have

$$\mathrm{NC} = \frac{102{,}704}{317{,}750} = 32.32\%$$

Comments: Most of the candidates who answered part b received full credit for this part.

d)

Under PUC the curve of contribution rates by age is less steep, so the NC at age 60 will be closer to the NC at age 35, implying that the change will be smaller. The reason is that the PUC pre-pays for future pay increases on accrued benefits, while the TUC does not. That means that TUC NC rates at older ages must pay for the new accrued benefit, and in addition must pay to upgrade all past accruals for the most recent pay rise. This creates a very steep curve of contribution rates at older ages.

Comments: The vast majority of candidates omitted the part.