GIRR Model Solutions
Fall 2021

1. **Learning Objectives:**

2. The candidate will demonstrate the ability to prepare claims and exposure data for general insurance actuarial work.

Learning Outcomes:

(2c) Calculate written, earned, in-force and unearned premiums for portfolios of policies with various policy terms and earnings patterns.

(2d) Adjust historical earned premiums to current rate levels.

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 11 and 12.

Commentary on Question:

This question tests the candidate's understanding of written premiums, earned premiums and unearned premiums. In addition, this question tests the candidate's understanding of adjusting premiums to current rate levels.

Solution:

(a) Calculate the 2020 calendar year total written premiums.

Policy#	2020 Written Premium	
1	2,205	written in 2020 (gets the 5% renewal increase)
2	1,440	written in 2020 (Feb 1 & Aug 1 renewal)
3	1,800	written in 2020
3	–600	Cancellation (4 months remaining at the time of cancellation)
Total	4,845	

1. Continued

(b) Calculate the 2020 calendar year total earned premiums.

Policy#	Period	Original Written Premium	# of Months Earned in 2020	2020 Earned Premium
1	Jan 1, 2020 to Oct 31, 2020	2,100	10	1,750.00
1	Nov 1, 2020 to Dec 31, 2020	2,205	2	367.50
2	Feb 1, 2020 to July 31, 2020	720	6	720.00
2	Aug 1, 2020 to Dec 31, 2020	720	5	600.00
3	April 1, 2020 to Nov 30, 2020	1,800	8	1,200.00
Total 2020 Earned Premium				4,637.50

Notes: 2,205 = 2,100×1.05
368 = 2,205×2/12

(c) Calculate the total unearned premiums as of December 31, 2020.

Policy#	Unearned Premium
1	1,837.50
2	120.00
3	0.00
Total	1,957.50

Notes: 1,837.50 = 2,205 – 367.50
Policy 3 not in force at the end of 2020, therefore unearned premium = 0

(d) Explain why the parallelogram approach would be inaccurate for this calculation.

Commentary on Question:
Policy duration is not relevant to the parallelogram approach being inaccurate.

Any two of the following are acceptable:

- The policies are not written evenly through the period.
- The parallelogram approach is an approximation method and with so few policies the actual calculation is more accurate.
- The parallelogram approach more appropriate for an entire book of business and not few individual policies.

1. Continued

(e) Calculate the 2020 total earned premiums adjusted to the current rate level.

Policy#	Period	2020 Earned Premium	Earned Premium at Current Rate Level
1	Jan 1, 2020 to Oct 31, 2020	1,750.00	1,837.50
1	Nov 1, 2020 to Dec 31, 2020	367.50	367.50
2	Feb 1, 2020 to July 31, 2020	720.00	756.00
2	Aug 1, 2020 to Dec 31, 2020	600.00	630.00
3	April 1, 2020 to Dec 1, 2020	1,200.00	1,260.00
Total 2020 Earned Premium at Current Rate Level			4,851.00

2. Learning Objectives:

3. The candidate will know how to calculate and evaluate projected ultimate values.

Learning Outcomes:

(3a) Identify considerations for selecting methods for estimating ultimate claims.
(3e) Describe the key assumptions underlying the following projection methods: development method, frequency-severity methods, expected method, Bornhuetter Ferguson method, Benktander method, Cape Cod method, Generalized Cape Cod, and Berquist-Sherman adjustments to the development method.
(3f) Demonstrate knowledge of good practice related to projecting ultimate values.
(3g) Estimate ultimate values using the methods cited in (3e).

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 14, 16 and 17.

Commentary on Question:

This question tests the candidate's understanding of the expected method. In addition, this question tests the calculation of ultimate claims using the expected and Bornhuetter Ferguson methods, as well as evaluating the reasonableness of the inputs used for the Bornhuetter Ferguson method.

Solution:

(a) Describe one advantage of using the pure premium approach to the expected method, rather than the claim ratio approach.

No adjustment is required for premium rate changes.

Or

It is best to choose an exposure base that requires no adjustment.

Or

It may be possible to select a pure premium exposure base that is a leading indicator of claims experience.

(b) Describe why reinsurers typically use the claim ratio approach to the expected method, rather than the pure premium approach.

Exposures are usually not available.

2. Continued

(c) Describe why reinsurers often use the expected method rather than the development method.

Reinsurance data is often subject to significant lags in immature years, which make development-based projections less reliable.

(d) Contrast the leveraged nature of cumulative development factors with the leveraged nature of trend factors.

Large development factors in immature periods can increase uncertainty.
Large trend factors in older/mature periods can increase uncertainty.

(e) Describe one approach the actuary may consider to moderate the leveraging effect of actuarial factors.

The actuary should consider excluding the oldest and most recent time periods when selecting averages based on historical experience.

Or

The actuary should consider excluding highly leveraged years (recent or old) from its selected experience period (either older or recent years).

(f) Calculate ultimate claims using the pure premium approach to the expected method.

Report Year (RY)	(1) Earned Dentists	(2) Actual Reported Claims	(3) Cumulative Development Factors
2013	12,603	12,974,000	1.042
2014	13,190	13,846,250	1.087
2015	13,631	14,074,250	1.149
2016	13,988	13,332,300	1.235
2017	15,364	14,057,100	1.351
2018	15,949	13,586,400	1.515
2019	16,270	12,601,600	1.754
2020	16,468	10,118,900	2.128

2. Continued

RY	(4) = (2)(3) Projected Ultimate Claims from Development Method	(5) = $1.03^{(2020-RY)}$ Trend	(6) = (4)(5)/(1) Trended Pure Premium	(7) = 1,357.30/(5) Detrended Pure Prem	(8) = (1)(7) Projected Ultimate based on Expected Method
2013	13,518,908.00	1.230	1,319.25	1,104	13,908,760
2014	15,050,873.75	1.194	1,362.51	1,137	14,993,275
2015	16,171,313.25	1.159	1,375.32	1,171	15,959,403
2016	16,465,390.50	1.126	1,324.85	1,206	16,868,706
2017	18,991,142.10	1.093	1,350.70	1,242	19,083,924
2018	20,583,396.00	1.061	1,369.17	1,279	20,404,881
2019	22,103,206.40	1.030	1,399.28	1,318	21,440,029
2020	21,533,019.20	1.000	1,307.57	1,357	22,351,975
Total	144,417,249.20				145,010,953
	Average Trended PP excluding 2020:				
		All Years	1,357.30		
		All Years excluding high/low	1,356.51		
		Latest 5	1,350.31		
		Selected	1,357.30		

(g) Calculate ultimate claims using the Bornhuetter Ferguson method, where the a priori expected claims are the estimated ultimate claims from the expected method in part (f).

Report Year	(9) = 1 – 1/(3) Expected % Unreported	(10) = (8)(9) Expected Unreported Claims	(11) = (2) + (10) Ultimate Claims
2013	4.0%	560,622	13,534,622
2014	8.0%	1,200,014	15,046,264
2015	13.0%	2,069,583	16,143,833
2016	19.0%	3,209,835	16,542,135
2017	26.0%	4,958,147	19,015,247
2018	34.0%	6,936,313	20,522,713
2019	43.0%	9,216,523	21,818,123
2020	53.0%	11,848,227	21,967,127
Total		39,999,264	144,590,064

2. Continued

(h) Evaluate the reasonableness of the inputs for the Bornhuetter Ferguson method in part (g).

Commentary on Question:
The key point is to test and conclude on the reasonability of the input assumptions. Under this particular scenario (comparing Expected Method and Bornhuetter Ferguson Method), the difference in method ultimate claims will be the same as the difference in actual versus expected. However, the percentage difference by year must still relate to expected claims, not ultimate claims, otherwise the variability in immature years will be minimized.

	(12) = (8) – (10)	(13) = (2) – (12)	(14) = (13)/(12)
Report Year	Expected Reported Claims	Difference Actual vs. Expected	Percentage Difference
2013	13,348,138	–374,138	–2.8%
2014	13,793,261	52,989	0.4%
2015	13,889,820	184,430	1.3%
2016	13,658,871	–326,571	–2.4%
2017	14,125,776	–68,676	–0.5%
2018	13,468,568	117,832	0.9%
2019	12,223,506	378,094	3.1%
2020	10,503,748	–384,848	–3.7%
Total	105,011,689	–420,889	–0.4%

The difference is reasonable in total. The largest difference is in the most recent two years, which is expected based on maturity.

3. Learning Objectives:

6. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

Learning Outcomes:

(6m) Describe key considerations in the analysis of deductible factors and increased limits factors.

(6n) Calculate deductible factors and increased limits factors.

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 33.

Commentary on Question:

This question tests the candidate's understanding of deductible factors and increased limit factors, including calculating elimination ratios and checking deductible factors for consistency.

Solution:

(a) Provide two reasons insurers use deductibles, other than to directly reduce the amount of claims paid.

Any two of the following is acceptable:

- Assist in reducing moral and morale hazard
- Encourage insureds to adhere to some measure of risk control
- Eliminate the processing costs associated with small claims
- Reduce exposure to catastrophic events

(b) Provide two reasons insurers use limits, other than to directly reduce the amount of claims paid.

Any two of the following is acceptable:

- To accommodate the financial needs and risk preferences of insureds
- To reflect the capacity of insurers
- To substitute for exclusions (in property policies)

(c) Explain why an analysis of increased limits factors is more likely to use a statistical distribution.

A limits analysis is working with the right tail of the distribution. Often times, there are not enough claims in the right tail of the distribution to credibly measure increased limits factors at higher limits. Therefore, an analysis of increased limits factors is more likely to use a statistical distribution.

3. Continued

(d) Determine the elimination ratios and deductible factors for each of the deductible options.

	Elimination Ratios by Accident Year		
Accident Year (AY)	250	500	1,000
2015	3.85%	8.04%	11.07%
2016	7.53%	12.92%	16.88%
2017	8.98%	14.40%	18.30%
2018	7.86%	13.32%	17.28%
2019	9.96%	15.75%	19.82%
2020	7.27%	12.37%	16.06%
All years average	7.58%	12.80%	16.57%
All years average excl. 2015	8.32%	13.75%	17.67%
Selected elimination ratio	8.32%	13.75%	17.67%
Deductible factor	0.9168	0.8625	0.8233

e.g.,

Elimination ratios for AY2015:

- 250: 3.85% = (1,128,906 – 1,085,419) / 1,128,906
- 500: 8.04% = (1,128,906 – 1,038,175) / 1,128,906
- 1,000: 11.07% = (1,128,906 – 1,003,976) / 1,128,906

Deductible factor for 250 deductible: 0.9168 = 1 – .0832

AY2015 seems to be an outlier, so the all years average excluding 2015 is selected.

(e) Evaluate the reasonability of the deductible factors calculated in part (d) using a consistency test.

Deductible	Deductible Factor	Marginal Rate Per 1,000
100	1.0000	
250	0.9168	0.5548
500	0.8625	0.2173
1000	0.8233	0.0782

e.g., $0.2667 = 1{,}000 \times \frac{(1.000 - .9618)}{(250 - 100)}$

Since the marginal rates are strictly decreasing, the deductible factors are reasonable.

4. Learning Objectives:

5. The candidate will understand trending procedures as applied to ultimate claims, exposures and premiums.

6. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

Learning Outcomes:

(5b) Identify the time periods associated with trending procedures.
(5e) Calculate trend factors for claims and exposures.
(6g) Calculate loadings for catastrophes and large claims.
(6h) Apply loadings for catastrophes and large claims in ratemaking.
(6j) Calculate indicated rates and indicated rate changes using the claim ratio and pure premium methods.

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 26, 30 and 31.

Commentary on Question:

This question tests the candidate's understanding of basic ratemaking, including the application of a loading for catastrophes in ratemaking.

Solution:

(a) Calculate the pure premium for the earthquake endorsement.

Midpoint of future rating period:	July 1, 2023
Exposure trend period (months): July 1, 2020 to October 1, 2020	3
Exposure trend = $(1.035^{(3/12)})$ =	1.00864
Severity trend period (months): October 1, 2020 to July 1, 2023	33
Severity trend = $(1.07^{(33/12)})$ =	1.20450
Trended modeled catastrophe claims = 225,000×1.009×1.204 =	273,352.46
Trended exposures = $15{,}000\times(1 + 0.035)^{((3 + 33)/12)}$ =	16,630.77
Pure premium = 273,352.46 / 16,630.77 =	16.44

(b) Calculate the premium for the earthquake endorsement.

Endorsement premium = (16.44 + 5) / (1 – 0.1 – 0.25) = 30.77

4. Continued

(c) Calculate the indicated rate for the basic homeowners coverage. Justify any selections.

Accident Year	(1) On Level Earned Premium (OLEP)	(2) Ultimate Claims	(3) Trend Period (years)	(4) Trended OLEP	(5) Trended Ultimate Claims	(6) = (5)/(4) Claim Ratio
2018	15,500,000	9,000,000	5	17,113,252	12,622,966	0.7376
2019	16,250,000	8,000,000	4	17,589,523	10,486,368	0.5962
2020	17,000,000	8,200,000	3	18,040,536	10,045,353	0.5568

Notes: (3) For 2020: July 1, 2020 to July 1, 2023 = 3 years
(4) For 2020: $18,040,536 = 17,000,000 \times 1.02^3$
(5) For 2020: $10,045,353 = 8,200,000 \times 1.07^3$

Selected claim ratio = 0.5765 (average of 2019 and 2020 is used as 2018 is an outlier)

Indicated rate change = 0.5765 / 0.57 – 1 = 0.0114
Indicated rate = 1,050×1.0114 = 1,061.97

(d) State whether you agree with management's proposal. Justify your response.

Commentary on Question:
Other justification responses are possible.

Do not agree.
Justification: There is additional administrative cost related to this optional add-on, such as the mid-term addition or cancellation.

5. Learning Objectives:

5. The candidate will understand trending procedures as applied to ultimate claims, exposures and premiums.

Learning Outcomes:

(5b) Identify the time periods associated with trending procedures.

(5c) Analyze and evaluate trend for claims (including frequency, severity, and pure premium) and exposures (including inflation-sensitive exposures and premiums).

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 26.

Commentary on Question:

This question tests the candidate's understanding of premium trend.

Solution:

(a) Provide two circumstances in which exposure and premium trend adjustments need to be considered for a ratemaking analysis.

- When working with inflation-sensitive exposures.
- Where a change (or shift) in the mix of exposures and rating characteristics results in a corresponding change in premiums over time.

(b) Calculate and select the annual premium trend due to the change in discount level. Justify your selection.

Calendar Year	5% discount	10% discount	Average Discount	Annual Change
2016	5.2%	9.3%	98.81%	
2017	5.0%	10.0%	98.75%	−0.06%
2018	4.5%	11.0%	98.68%	−0.08%
2019	4.5%	12.0%	98.58%	−0.10%
2020	6.5%	25.0%	97.18%	−1.42%
Average excluding 2020:				−0.08%

e.g., $98.81\% = 1 - 0.05 \times 0.052 - 0.1 \times 0.093$

$-0.06\% = 98.75\% / 98.81\% - 1$

Select −0.08% as the annual premium trend. Justification is that 2020 should be excluded as this is assumed to be a one-time change and the annual change should therefore return to historical levels after 2020.

5. Continued

(c) Calculate the premium trend factor to be used for 2018 using earned premium for the trending analysis and incorporating the annual trend selected in part (b).

2018 average earned date	July 1, 2018
2020 average earned date	July 1, 2020
Trending Period 1	24 months
Effective date of new rates	February 1, 2022
Average earned date of forecast period	February 1, 2023
Trending Period 2 (July 1, 2020 to February 1, 2023)	31 months

Trending factor period 1: $((1 - 0.0008)(1 + 0.0075))^{(24/12)} = 1.01345$
Trending factor period 2: $((1 - 0.002)(1 + 0.0075))^{(31/12)} = 1.01423$
Premium trend factor = $1.01345 \times 1.01423 = 1.02787$.

6. Learning Objectives:

1. The candidate will understand the key considerations for and key concepts underlying general insurance actuarial work.

3. The candidate will know how to calculate and evaluate projected ultimate values.

Learning Outcomes:

(1d) Understand the components of ultimate values.
(1f) Demonstrate the importance of understanding key terminology and interrelationships.
(3g) Estimate ultimate values using the methods cited in (3e).
(3j) Evaluate and justify selections of ultimate values based on the methods cited in (3e).

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 3, 15 and 21.

Commentary on Question:

This question tests the candidate's understanding of the evaluation and selection of estimated IBNR under various circumstances.

Solution:

(a) Describe what an *actuarial central estimate* represents according to U.S. ASOPs.

An actuarial central estimate represents an expected value over the range of reasonably possible outcomes.

(b) Assess the validity of the following statement:

"Credibility is not utilized in projecting unpaid claims for reserving."

Invalid; credibility is often reflected implicitly when projecting ultimate claims.

6. Continued

(c) Calculate the indicated IBNR as of December 31, 2020 for each of the frequency-severity method projections above.

	(1)	(2)	(3)
Accident Year	Reported Claims	Indicated IBNR Development Based	Indicated IBNR Claim Closure
2015	5,051,008	2,154	2,479
2016	5,453,150	55,306	53,536
2017	5,764,966	136,626	102,293
2018	5,967,139	275,802	337,862
2019	6,294,143	531,932	761,852
2020	5,980,004	1,173,792	1,398,061

Notes: (1) = (Earned Premium)(Reported Claim Ratio Triangle Latest Diagonal)
e.g., 2017: 5,764,966 = 8,669,122×66.5%
(2) = Ultimate Claims – (1)
e.g., 2017: 136,626 = 5,901,592 – 5,764,966
(3) = Ultimate Claims – (1)
e.g., 2017: 102,293 = 5,867,259 – 5,764,966

(d) Critique the appropriateness of each method as a potential IBNR selection for accident year 2018.

(i) Paid development method

(ii) Reported development method

(iii) Paid Bornhuetter Ferguson method

(iv) Reported Bornhuetter Ferguson method

(i) Paid development is not appropriate because it is under-responsive to large claim.

(ii) Reported development is not appropriate because it is over-responsive to large claim.

(iii) Paid Bornhuetter Ferguson is not appropriate because it is under-responsive to large claim.

(iv) Reported Bornhuetter Ferguson is an appropriate method because it is not distorted by large claim and also recognizes relative immaturity of a liability coverage.

7. Learning Objectives:

6. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

Learning Outcomes:

(6l) Calculate risk classification changes.

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 32.

Commentary on Question:

This question tests the candidate's understanding of classification ratemaking.

Solution:

(a) Describe the following objectives with respect to risk classification:

(i) fair or equitable premium rates

(ii) socially adequate premium rates

(i) Fair premium means the rates that reflect expected cost of the insured.

(ii) Socially adequate premium means the rates that reflect the affordability of the insurance considering each person's economic situation.

(b) Describe how the objectives in part (a) could be contradictory when rating by age for drivers aged 18, 40, and 80 in automobile insurance.

Age 18: Typically, a high premium for younger individuals, but this may not be viewed as socially adequate depending on the jurisdiction.

Age 40: Although premiums calibrated to be fair may be deemed socially adequate at this age, there may be subsidies given the socially adequate concerns for younger (18) and older (80) individuals, leaving these premiums no longer at fair prices.

Age 80: Fair premiums would tend to be higher than socially adequate premiums for this age group.

7. Continued

(c) Assess the validity of the following statement:

"Determining the expected costs for a particular class of risks is the same as predicting the costs for an individual risk in the class."

This is not valid as it is impossible and unnecessary to predict costs for any individual risk.

(d) Describe two operational considerations that affect the *practicality* of designing and maintaining a risk classification system.

Any two of the following are acceptable:

- Objectivity: Where possible, the evaluation of a risk characteristic should be factual and not judgmental.
- Cost: One should note that costs arise from obtaining, storing, and analyzing the data required for actuarial work supporting the risk classification system.
- Verifiablity: The risk characteristics used in a risk classification system should be reliable and conveniently verifiable. Characteristics such as age, gender, and occupation can, in general, be reliably measured.

8. Learning Objectives:

2. The candidate will demonstrate the ability to prepare claims and exposure data for general insurance actuarial work.

4. The candidate will understand financial reporting of claim liabilities and premium liabilities.

Learning Outcomes:

(2c) Calculate written, earned, in-force and unearned premiums for portfolios of policies with various policy terms and earnings patterns.

(4g) Describe the components of premium liabilities in the context of financial reporting.

(4h) Evaluate premium liabilities.

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 11 and 24.

Commentary on Question:

This question tests the candidate's understanding of unearned premiums and premium liabilities.

Solution:

(a) Calculate unearned premium by line of business as of December 31, 2020.

	(1)	(2) = (1)/12	(3)	(4)
			Unearned Premiums (UEP) by Quarter	
Underwriting Quarter	Months Remaining at Dec. 31, 2020	Unearned Proportion	Auto	Homeowners
2020Q1	1.5	0.125	26,250.00	40,000.00
2020Q2	4.5	0.375	75,187.50	121,875.00
2020Q3	7.5	0.625	123,437.50	206,250.00
2020Q4	10.5	0.875	179,462.50	281,750.00
Unearned Premiums at Dec. 31, 2020			404,337.50	649,875.00

Notes: (3) = (2)×(Auto Written Premiums)
(4) = (2)×(Homeowners Written Premiums)

8. Continued

(b) Calculate the equity in unearned premiums as of December 31, 2020 by line of business.

Homeowners expected claims need to be calculated by quarter; auto does not since expected claim ratios are the same for each quarter.

2021 Unexpired Months Allocated to Accident Quarter

Underwriting Quarter	Q1	Q2	Q3	Q4	Total Months Unexpired
2020Q1	1.5				1.5
2020Q2	3	*1.5*			*4.5*
2020Q3	3	3	1.5		7.5
2020Q4	3	3	3	1.5	10.5

2021 Unearned Premiums Allocated to Accident Quarter

Underwriting Quarter	Q1	Q2	Q3	Q4	Total
2020Q1	40,000				40,000
2020Q2	81,250	*40,625*			121,875
2020Q3	82,500	82,500	41,250		206,250
2020Q4	80,500	80,500	80,500	40,250	281,750
Total UEP	284,250	203,625	121,750	40,250	649,875
Gross Expected Claim Ratios	70%	70%	80%	70%	
Expected Claims	198,975	142,538	97,400	28,175	467,088

e.g., 2020Q2 @ 2021 Q2: 40,625 = 121,875×1.5/4.5
Expected claims for Q2: 142,538 = 203,625×70%

Auto expected claims = 72%×404,337.50 = 291,123

		Auto	Homeowners
(1)	Unearned premiums	404,337.50	649,875.00
(2)	Expected claims	291,123.00	467,087.50
(3)	ULAE (7.5%×(2) for Auto, 10%×(2) for Homeowners)	21,834.23	46,708.75
(4)	Maintenance expenses (5%×(1))	20,216.88	32,493.75
(5)	Net premium liabilities ((2) + (3) + (4))	333,174.10	546,290.00
(6)	Equity/(Deficiency) ((1) – (5))	71,163.40	103,585.00

8. Continued

(c) Describe two potential implications of this result.

- A premium deficiency reserve may be required for the company.
- General Liability rates appear to be inadequate and should be reviewed.

9. **Learning Objectives:**

3. The candidate will know how to calculate and evaluate projected ultimate values.

7. The candidate will understand the need for monitoring results.

Learning Outcomes:

(3e) Describe the key assumptions underlying the following projection methods: development method, frequency-severity methods, expected method, Bornhuetter Ferguson method, Benktander method, Cape Cod method, Generalized Cape Cod, and Berquist-Sherman adjustments to the development method.

(3g) Estimate ultimate values using the methods cited in (3e).

(7b) Analyze actual claims experience relative to expectations.

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 18 and 36.

Commentary on Question:

This question tests the calculation of ultimate claims using the Cape Cod method. This question also tests the candidate's understanding of monitoring actual versus expected reported claims.

Solution:

(a) Describe why an exposure base that is not inflation-sensitive is preferred over an exposure base that is inflation-sensitive.

The exposure base that requires the least adjustment is preferred because additional adjustments add imprecision to the projection process.

9. Continued

(b) Derive a selected adjusted expected pure premium.

Accident Year (AY)	(1) Earned Exposures	(2) Reported Claims as of Dec. 31, 2020	(3) Cumulative Development Factors	(4) = 1/(3) Expected % Developed
2012	8,391	1,002	1.008	99.2%
2013	8,402	1,045	1.020	98.0%
2014	8,788	1,216	1.038	96.3%
2015	9,088	664	1.063	94.0%
2016	9,325	710	1.097	91.1%
2017	9,704	593	1.146	87.3%
2018	10,073	739	1.227	81.5%
2019	10,339	632	1.432	69.8%
2020	10,591	448	2.148	46.6%
Total	84,701	7,049		

e.g., Column (3) for 2015: $1.063 = 1.008 \times 1.012 \times 1.018 \times 1.024$

AY	(5) = (1)(4) Used-Up On-Level Exposures	(6) = $0.99^{(2020-AY)}$ Pure Premium Trend	(7) Tort Reform	(8) = (2)(6)(7) Adjusted Claims at Dec. 31, 2020
2012	8,324	0.923	0.950	878
2013	8,236	0.932	0.950	925
2014	8,463	0.941	0.950	1,088
2015	8,546	0.951	0.950	600
2016	8,497	0.961	0.950	648
2017	8,470	0.970	0.950	547
2018	8,209	0.980	1.000	724
2019	7,220	0.990	1.000	626
2020	4,931	1.000	1.000	448
Total	70,897			6,484

Adjusted expected pure premium = 6,484 / 70,897 = 0.0915

9. Continued

(c) Derive projected ultimate claims.

AY	(9) = 0.0915×(1)/[(6)(7)] Expected Claims (Ultimate)	(10) = 1 - (4) Expected % Undeveloped	(11) = (9)(10) Expected Unreported	(12) = (2) + (11) Projected Ultimate Claims
2012	875	0.8%	7	1,009
2013	868	2.0%	17	1,062
2014	899	3.7%	33	1,249
2015	920	6.0%	55	719
2016	934	8.9%	83	793
2017	963	12.7%	122	715
2018	940	18.5%	174	913
2019	955	30.2%	288	920
2020	969	53.4%	518	966
Total	8,322		1,297	8,346

(d) Calculate the difference between the expected reported claims underlying the Cape Cod calculations in part (c) and actual reported claims as of December 31, 2020.

AY	(2) Reported Claims as of Dec. 31, 2020	(13) = (9) - (11) Expected Reported Claims	(14) = (2) - (13) Difference Actual vs. Expected
2012	1,002	868	134
2013	1,045	851	194
2014	1,216	865	351
2015	664	865	(201)
2016	710	852	(142)
2017	593	840	(247)
2018	739	766	(27)
2019	632	667	(35)
2020	448	451	(3)
Total	7,049	7,025	24

9. Continued

(e) Describe two other possible circumstances that could cause an anomaly as shown above.

Commentary on Question:
Other possible circumstances are possible.

Any two of the following are acceptable:

- Development may be lower in recent years due to operational changes or changes in experience.
- Experience may have improved beginning in AY 2016. Maybe frequency or severity improved due to loss prevention or loss control activities.
- A policy change may have been made in 2016 which reduced claim exposure (e.g., higher deductible).

10. Learning Objectives:

6. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

Learning Outcomes:

(6r) Calculate rates for claims-made coverage as well as claims-made maturity and tail factors.

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 34.

Commentary on Question:

This question tests the understanding of claims-made ratemaking.

Solution:

(a) State one advantage and one disadvantage of claims-made coverage from an insurer's perspective.

Advantage: more predictable loss cost
Disadvantage: less opportunity for investment income (or have to offer tail policy)

(b) Demonstrate, with a numerical example, a situation in which the claims-made loss cost is greater than the occurrence loss cost.

Commentary on Question:

Other solutions are possible.

Example:
Consider the case where the reporting period is two years with a reporting pattern of 50% in year 1 and 50% in year 2. Assume claims cost trend is –20%. For an occurrence claims cost of 100, the claims-made claims cost would be $50 \times \left(1 + \frac{1}{1 - 0.20} \right) = 112.50$. Thus, the claims-made claims cost is greater.

10. Continued

(c) Calculate tail factors for a claims-made policy for the following maturities:

(i) Second-year

(ii) Mature

	Report Year			
AY Lag	1	2	3	4
0	**0.4**	0.4	0.4	0.4
1	**0.2**	***0.2***	0.2	0.2
2	0.2	***0.2***	***0.2***	0.2
3	0.2	0.2	***0.2***	***0.2***

(i) Second-year tail factor = (***0.2*** + ***0.2*** + ***0.2*** + ***0.2*** + ***0.2***) / (**0.4** + **0.2**) = 1.667

	Report Year			
AY Lag	1	2	3	4
0	**0.4**	0.4	0.4	0.4
1	**0.2**	***0.2***	0.2	0.2
2	**0.2**	***0.2***	***0.2***	0.2
3	**0.2**	***0.2***	***0.2***	***0.2***

(ii) Mature tail factor = (***0.2*** + ***0.2*** + ***0.2*** + ***0.2*** + ***0.2*** + ***0.2***) / (**0.4** + **0.2** + **0.2** + **0.2**) = 1.2

(d) Calculate CM's earned premium for 2021, 2022 and 2023 for a mature tail policy effective January 1, 2021 with a premium of 25,000.

With a 25,000 tail premium split into six units, the earning would be as follows:
2015: (3/6) of 25,000 = 12,500
2016: (2/6) of 25,000 = 8,333.33
2017: (1/6) of 25,000 = 4,116.67

11. Learning Objectives:

3. The candidate will know how to calculate and evaluate projected ultimate values.

Learning Outcomes:

(3e) Describe the key assumptions underlying the following projection methods: development method, frequency-severity methods, expected method, Bornhuetter Ferguson method, Benktander method, Cape Cod method, Generalized Cape Cod, and Berquist-Sherman adjustments to the development method.

(3g) Estimate ultimate values using the methods cited in (3e).

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 14.

Commentary on Question:

This question tests the candidate's understanding of the development-based frequency-severity method for estimating ultimate claims.

Solution:

(a) Calculate the ultimate claims for accident year 2020 using the development-based frequency-severity method. Justify any selections.

	(1)	(2)	(3)	(4)	(5)	(6) = (1)(3)(5)	(7) = (2)(4)
Accident Year (AY)	Frequency	Severity	Frequency Trend @0.5%	Severity Trend @4.7%	Change from Court Ruling	Trended Frequency	Trended Severity
2014	0.0424	28,830	1.0304	1.3173	1.06	0.0463	37,977
2015	0.0427	30,014	1.0253	1.2582	1.06	0.0464	37,762
2016	0.0429	31,554	1.0202	1.2017	1.06	0.0464	37,917
2017	0.0431	32,987	1.0151	1.1477	1.06	0.0464	37,860
2018	0.0436	34,257	1.0100	1.0962	1.06	0.0467	37,553
2019	0.0435	36,098	1.0050	1.0470	1.06	0.0463	37,795
2020	0.0452	37,317	1.0000	1.0000	1.00	0.0457	37,317
					All years average	0.0463	37,740
					Average excluding 2020	0.0464	37,811

e.g., (1) for 2020: 0.452 = 431 / 9,542
(2) for 2020: 37,317 = 16,270,027 / 431

Selected frequency = 0.0463 (both averages account for the court ruling change, so either is reasonable)
Selected severity = 37,740 (no outliers and no significant trend, therefore all years average is reasonable)

11. Continued

Ultimate claims = 0.0463×9,542×37,740 = 16,680,290.

(b) Calculate the percentage growth in accident year 2020 IBNR in changing from the development method to the development-based frequency-severity method.

IBNR from development method = 16,270,027 – 5,778,161 = 10,491,866
IBNR from F-S method = 16,680,290 – 5,778,161 = 10,902,129
Percent growth = 10,902,129 / 10,491,866 – 1 = 3.91%%

(c) Explain why the accident year 2020 IBNR calculated using the development-based frequency-severity method is likely to be more appropriate than the IBNR calculated using the development method.

Any two of the following are acceptable:

- If we are confident in the expected increase in claim frequency for AY 2020 then the F-S method is more likely to be appropriate.
- Development method does not adjust for AY 2020 expected increase in claim frequency.
- This is seen by the fact the F-S method ultimate claims are 2.5% higher than the development method ultimate claims, and the F-S method IBNR is 3.91% higher than the development method IBNR.

12. Learning Objectives:

3. The candidate will know how to calculate and evaluate projected ultimate values.

Learning Outcomes:

(3f) Demonstrate knowledge of good practice related to projecting ultimate values.
(3h) Explain the effect of changing conditions on the projection methods cited in (3e).
(3i) Assess the appropriateness of the projection methods cited in (3e) in varying circumstances.

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 20 and 21.

Commentary on Question:

This question tests the evaluation of the reasonableness of the various methods of projecting ultimate claims under specific circumstances as well as under changing conditions.

Solution:

(a) Describe two diagnostics that can be used to confirm the reasonableness of projected ultimate claims.

Any two of the following are acceptable:

- Claim ratio: ultimate claims divided by earned premiums
- Severity: ultimate claims divided by ultimate counts
- Pure Premium: ultimate claims divided by earned exposures
- IBNR: ultimate claims divided by earned exposures
- Total unpaid claims: ultimate claims less paid claims
- Average IBNR: IBNR divided by IBNR counts
- Average unpaid claims: total unpaid claims divide by the sum or open and IBNR counts

(b) Explain what effect the tort reform is likely to have on reported claim development factors if the data is organized as follows:

(i) On an accident year basis.

(ii) On a report year basis.

(i) Development factors in latest two calendar years (i.e., diagonals) will decrease.

(ii) Development factors in latest two report years (i.e., rows) will decrease.

12. Continued

(c) Recommend a preferred approach to estimating ultimate claims for each scenario in part (b). Justify your recommendation.

Commentary on Question:
Other possible approaches are possible.

(i) Use a Berquist-Sherman adjustment to adjust historical triangle data to be consistent with the average severity in latest two calendar years.

(ii) Use a frequency-severity method and adjust the severity in the latest two report years to reflect the cap.

(d) Assess the appropriateness of each selection (i) to (iv).

(i) AY2013, Bornhuetter Ferguson method using reported claim ratio data: The Bornhuetter Ferguson method is most appropriate for immature years. Also, reported data may be distorted by the change in case adequacy. Conclusion: Not appropriate.

(ii) AY2016, Cape Cod method using paid claim data: The Cape Cod method is more appropriate for immature years, however, paid cumulative development factors still show some unpaid, so Cape Cod method is reasonable. Also, the paid data is not distorted by the change in case adequacy. Conclusion: Appropriate.

(iii) AY2019, Development method using paid claim data: Paid data is good to use, but the 2019 cumulative development factor is too highly leveraged to be reliable. Conclusion: Not appropriate.

(iv) AY2020, Expected method using reported claim ratio data: This method uses a priori data, which is not distorted by change in case adequacy. This method is good for immature accident years. Conclusion: Appropriate.

13. Learning Objectives:

3. The candidate will know how to calculate and evaluate projected ultimate values.

Learning Outcomes:

(3k) Estimate ultimate claims by layer using common methods.

(3l) Understand the differences in development patterns and trends for various claim layers.

Sources:

Fundamentals of General Insurance Actuarial Analysis 2019 Supplement, J. Friedland, Appendix I.

Commentary on Question:

This question tests the candidate's understanding of claims excess of a limit.

Solution:

(a) Show that the year-to-year increase in ultimate claims in the layer 1,000,000 excess of 500,000 is 4.3% for this sample.

	Year 1		Year 2	
Claim #	Ground-Up Ultimate Claims	Claims in the Layer	Ground-Up Ultimate Claims	Claims in the Layer
1	495,000	0	519,750	19,750
2	525,000	25,000	551,250	51,250
3	1,200,000	700,000	1,260,000	760,000
4	1,490,000	990,000	1,564,500	1,000,000
5	1,800,000	1,000,000	1,890,000	1,000,000
Total	5,510,000	2,715,000	5,785,500	2,831,000

Year-to-year increase in ultimate claims in the layer 1,000,000 excess of 500,000 = 2,831,000 / 2,715,000 – 1 = 4.3%.

13. Continued

(b) Explain how the annual claim trend for ultimate claims in the layer 1,000,000 excess of 500,000 could be *greater* than 5% for the entire book of business.

Commentary on Question:
The example provided here is for demonstration purposes and is not needed for full credit.

This could occur if there are more claims at the lower limits (between 500,000/1.05 and 500,000) relative to claims greater than or equal to 1,500,000/1.05 in the entire book of business.

For example, if claims #4 and #5 are removed from the part (a) example , the trend is:

831,000 / 725,000 – 1 = 14.6%.

(c) Calculate the trended ultimate claims in the layer 1,000,000 excess of 500,000 for ratemaking purposes, using theoretically-derived cumulative development factors.

CDF 500,000 = 1.25×0.78/0.72 = 1.154
CDF 1,500,000 = 1.25×0.95/0.91 = 1.197

Trend period = average accident date in 2020 to average accident date in future rating period = July 1, 2020 to April 1, 2023 = 33 months, or 2.75 years.

Limit/Layer	Reported Claims	CDF	Trend Factor	Trended Ultimate Claims
500,000 limit	6,500,000	1.154	1.0992	8,244,175
1,500,000 limit	8,200,000	1.197	1.1376	11,169,537
In the layer 1,000,000 excess of 500,000				2,925,362

e.g., $1.0992 = 1.035^{2.75}$
8,244,175 = 6,500,000×1.0992
2,925,362 = 11,169,537 – 8,244,175

14. Learning Objectives:

3. The candidate will know how to calculate and evaluate projected ultimate values.

4. The candidate will understand financial reporting of claim liabilities and premium liabilities.

Learning Outcomes:

(3d) Analyze development triangles for investigative testing.
(3g) Estimate ultimate values using the methods cited in (3e).
(4f) Calculate claim liabilities.

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 13, 19, and 23.

Commentary on Question:

This question tests the candidate's understanding of Berquist-Sherman adjustments when there has been a change in case estimate adequacy.

Solution:

(a) Calculate the average case estimate triangle.

Accident	Average Case Estimates					
Year (AY)	12	24	36	48	60	72
2015	5,050	14,289	28,750	39,278	41,527	22,216
2016	5,265	14,852	29,973	42,364	53,583	
2017	5,504	15,689	32,047	49,963		
2018	5,718	16,944	38,052			
2019	6,098	21,539				
2020	7,324					

e.g., AY 2018 at 24 months: $16{,}944 = \dfrac{(7{,}385{,}262 - 2{,}522{,}316)}{(1{,}993 - 1{,}706)}$

(b) Explain why the average case estimate triangle indicates reducing, increasing or stable case reserve adequacy.

Commentary on Question:

Increasing down each column is not an indication of increasing case adequacy, as average case estimates are expected to increase down each column at the rate of trend in a stable environment.

The last diagonal is much higher than previous years, suggesting increasing case reserve adequacy.

14. Continued

(c) Calculate IBNR by accident year using the reported development method, with a Berquist-Sherman adjustment.

Adjusted Average Case = Last Diagonal from part (a), trended to each AY at 5.9%:

AY	12	24	36	48	60	72
2015	5,499	17,125	32,040	44,551	50,598	22,216
2016	5,823	18,136	33,930	47,180	53,583	
2017	6,167	19,206	35,932	49,963		
2018	6,530	20,339	38,052			
2019	6,916	21,539				
2020	7,324					

e.g., 6,916 = 7,324 / 1.059

Adjusted Case Estimates = Adjusted Average Case Estimate × Open Counts:

AY	12	24	36	48	60	72
2015	3,546,599	5,154,681	5,286,561	5,435,229	3,541,866	888,653
2016	4,338,150	6,147,959	5,666,328	5,850,267	3,911,586	
2017	4,748,265	7,355,734	6,934,873	6,395,285		
2018	4,486,389	5,837,207	6,773,251			
2019	4,460,627	7,301,614				
2020	5,302,378					

e.g., 4,460,627 = 6,916 × (1,847 – 1,202)

Adjusted Reported Claims = Paid Claims + Adjusted Case Estimates

AY	12	24	36	48	60	72
2015	4,264,284	7,556,451	9,675,018	12,476,031	13,549,178	13,597,939
2016	5,082,091	8,641,763	10,236,617	13,210,469	14,395,391	
2017	5,478,569	9,882,692	11,552,773	13,826,396		
2018	5,229,169	8,359,523	11,327,707			
2019	5,167,376	9,902,568				
2020	6,046,806					

14. Continued

Commentary on Question:
Other age-to-age development factor selections are possible.

Development Factors:

AY	12-24	24-36	36-48	48-60	60-72	72 to Ult.
2015	1.772	1.280	1.290	1.086	1.004	
2016	1.700	1.185	1.291	1.090		
2017	1.804	1.169	1.197			
2018	1.599	1.355				
2019	1.916					
2020						
Weighted average	1.758	1.242	1.256	1.088	1.004	
Age-to-Ultimate	3.055	1.738	1.399	1.114	1.024	1.020

	(1)	(2)	(3) = (1)(2)	(4) = (3) – (1)
AY	Reported Claims	Age-to-Ultimate Development Factor	Ultimate Claims	IBNR
2015	13,597,939	1.020	13,869,898	271,959
2016	14,395,391	1.024	14,736,142	340,751
2017	13,826,396	1.114	15,397,911	1,571,515
2018	11,327,707	1.399	15,842,149	4,514,442
2019	9,902,568	1.738	17,207,395	7,304,827
2020	6,046,806	3.055	18,473,435	12,426,629
Total	69,096,807		95,526,929	26,430,122

15. Learning Objectives:

6. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

Learning Outcomes:

(6j) Calculate indicated rates and indicated rate changes using the claim ratio and pure premium methods.

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 31.

Commentary on Question:

This question tests the candidate's understanding of basic ratemaking as well as forecasting profit.

Solution:

(a) Demonstrate that the indicated rate change using the pure premium approach is 5.9%.

Commentary on Question:

The pure premium approach is required for this part.

Average earned premium at current rate level = 8,100,000×1.030×1.007/11,000 =	763.76
Trended ultimate claims = 0.78×11,000×763.76 =	6,553,093
Trended pure premium = 6,553,093/11,000 =	595.74
Total fixed expenses = 0.05×763.76 =	38.19
Indicated rate = (595.74×(1 + 0.09) + 38.19) / (1 – 0.10 – 0.05) =	808.87
Indicated rate change = 808.87 / 763.76 – 1 =	5.91%

15. Continued

(b) Calculate the forecasted profit per policy for policies written in 2022, 2023, 2024 and 2025.

Commentary on Question:
Use the equation: Premiums = Claims + Expenses + Profit and Contingencies, to compare the per policy charged premium to the per policy expenses plus claims each year to solve for the profit per policy.

Calendar Year	(1) Required Premium	(2) = (1)×0.05 Fixed Expenses	(3) = (1)×0.1 Variable Expenses	(4) PP with ULAE	(5) Charged Premium	(6) Profit
2022	808.87	40.44	80.89	649.35	794.31	23.63
2023	812.11	40.61	81.21	662.39	794.31	10.11
2024	815.35	40.77	81.54	675.69	794.31	–3.67
2025	818.62	40.93	81.86	689.25	794.31	–17.73

Notes:
(1) Required premium for 2023 = 808.87×1.004 (increase with premium trend each year)
(4) PP with ULAE for 2023 = 595.74×1.09×1.015×1.005 (increase with frequency and severity trend each year)
(5) 763.76×1.04
(6) = (5) – (2) – (3) – (4)

16. Learning Objectives:

2. The candidate will demonstrate the ability to prepare claims and exposure data for general insurance actuarial work.

3. The candidate will know how to calculate and evaluate projected ultimate values.

Learning Outcomes:

(2a) Create development triangles of claims and counts from detailed claim transaction data.

(3c) Identify the types of development triangles that can be used for investigative testing.

(3d) Analyze development triangles for investigative testing.

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 10 and 13.

Commentary on Question:

This question tests the candidate's understanding of creating a development triangle from detailed claims transaction data, identifying potential issues with data triangles, and diagnostic tests that can be used on data triangles.

Solution:

(a) Define "maturity age" in the context of a claim development triangle.

The maturity age refers to the time interval from the beginning of the experience period to the valuation date of the claims.

(b) Construct a development triangle of cumulative reported claims, by accident year, with maturity ages 6, 12, 18, 24, 30 and 36 months.

Accident Year (AY)	Incremental Paid Claims at Maturity Age (in Months)					
	6	12	18	24	30	36
2018	50	100	250	0	55	75
2019	265	0	30	185		
2020	0	275				

e.g., AY2019 at 6 months: 265 = 190 + 75

16. Continued

	Cumulative Paid Claims at Maturity Age (in Months)					
AY	6	12	18	24	30	36
2018	50	150	400	400	455	530
2019	265	265	295	480		
2020	0	275				

e.g., AY2019 at 18 months: 295 = 265 + 0 + 30

	Case Estimates at Maturity Age (in Months)					
AY	6	12	18	24	30	36
2018	150	200	75	390	410	350
2019	35	260	225	0		
2020	550	65				

e.g., AY2019 at 12 months: 260 = 35 + 225 + 0

	Reported Claims at Maturity Age (in Months)					
AY	6	12	18	24	30	36
2018	200	350	475	790	865	880
2019	300	525	520	480		
2020	550	340				

Reported claims = Cumulative paid claims + Case estimates
e.g., AY2019 at 12 months: 525 = 265 + 260

(c) Select which line of business was the likely source for each of the following claims, providing a justification for each selection:

(i) Claim 2 is likely Automobile physical damage as it has a short reporting delay and was settled within 6 months of claim occurrence.

(ii) Claim 3 is likely Medical malpractice claim as it has a long reporting delay and has not closed within 36 months of its occurrence.

(iii) Claim 7 is likely Workers' compensation claim as it was reopened after its initial settlement.

16. Continued

(d) Identify two anomalies relating to this triangle.

Any two of the following are acceptable:

- Reported pure premiums decreased in AYs 2015-2016, then increased again in AYs 2017 and subsequent accident years.
- Reported pure premium for AY2014 increased significantly at 72 months, then decreased again at 84 months.
- Reported pure premium development is increasing over time (i.e., development factors increase down each column).

(e) Describe a business, operational, or environmental change that could cause each of the anomalies identified in part (d).

Commentary on Question:
Only one change is needed for each anomaly identified in part (d).

- Reported pure premiums decreased in AYs 2015-2016, then increased again in AYs 2017 and subsequent accident years:
    - Changes in policy terms (e.g., limits, deductibles) could cause PP to change over time.
    - Changes in the type of insureds (exposures) could cause PP to change over time.
- Reported pure premium for AY2014 increased significantly at 72 months, then decreased again at 84 months:
    - The reporting of a large claim (or case estimate) which then decreased/normalized could cause an increase, then decrease in reported pure premiums.
    - The reporting of a large claim, which was subsequently covered by reinsurance (or subrogation) could cause an increase, then decrease in reported pure premiums.
- Reported pure premium development is increasing over time (i.e., development factors increase down each column).
    - Change in policy terms (e.g., limits, deductibles) could cause development to change over time.
    - Change in the type of insureds (exposures) could cause development to change over time.
    - Change in case reserve adequacy (or claim settlement patterns) could cause development to change over time.

17. Learning Objectives:

9. The candidate will understand the nature and application of catastrophe models used to manage risks from natural disasters.

Learning Outcomes:

(9a) Describe the structure and modules of catastrophe models.
(9b) Apply catastrophe modeling results in ratemaking, loss mitigation, risk selection, and reinsurance.
(9d) Understand and apply common risk metrics associated with catastrophe modeling results.

Sources:

Uses of Catastrophe Model Output, American Academy of Actuaries, July 2018.

Commentary on Question:

This question tests the candidate's understanding of catastrophe modeling.

Solution:

(a) Calculate the probability of reaching an amount of loss that activates reinsurance coverage for each of the reinsurance layers.

1/100 = 1.0%, for layer 1 (insured losses between the 100-year and 250-year PMLs from these two perils)

1/250 = 0.4%, for layer 2 (insured losses between the 250-year and 250-year PMLs from these two perils)

(b) Calculate Primary's reinsurance recoverables from this catastrophic event for each of the two layers.

Amount in 000s:
Layer 1 losses from 664,515 to 1,089,697
Layer 2 losses from 1,089,697 to 1,605,179

Insured loss total = 1,098,085 + 132,325, = 1,230,410

Layer 1 losses = min(1,089,697,1,230,410) – 664,515 = 425,182
Layer 2 losses = min(1,605,179,1,230,410) – 1,089,697 = 140,713

17. Continued

(c) Estimate Primary's reinsurance premium for each layer of coverage.

Amount in 000s:

	Layer 1	Layer 2
AAL	661	233
Risk load	5,838	3,718
Expense load	2,052	1,248
Premium	8,551	5,199

e.g., Risk load for layer 1: $5{,}838 = 0.85 \times 6{,}868$

Expense load for layer 1:

$$\frac{(661+5838)}{1-0.24} - (661+5838)$$

Premium for layer 1: $8{,}551 = 661 + 5{,}838 + 2{,}052 + 8{,}551$

(d) Provide two reasons why Primary should not calculate the total reinsurance premium using the underwriter's recommendation.

If ABC wants to cover insured losses in the layer between the 100-year and 500-year PMLs, it must get the combined perils PMLs because PMLs are not additive.

Premiums cannot be added because this will overstate the risk load due to the fact that the SDs are not additive. The combined perils SD is less than the sum of the SDs for all the perils covered.

18. Learning Objectives:

1. The candidate will understand the key considerations for and key concepts underlying general insurance actuarial work.

3. The candidate will know how to calculate and evaluate projected ultimate values.

4. The candidate will understand financial reporting of claim liabilities and premium liabilities.

Learning Outcomes:

(1d) Understand the components of ultimate values.
(3f) Demonstrate knowledge of good practice related to projecting ultimate values.
(3g) Estimate ultimate values using the methods cited in (3e).
(4a) Describe the key assumptions underlying ratio and count-based methods for estimating unpaid unallocated loss adjustment expenses.
(4b) Estimate unpaid unallocated loss adjustment expenses using ratio and count-based methods.

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 3, 14, and 22.

Commentary on Question:

This question tests the candidate's understanding of selecting development factors and estimating a tail factor using Boor's algebraic method. It also tests the calculation of unpaid ULAE using the classical paid-to-paid method, as well as an understanding of the Kittel refinement to the classical paid-to-paid method and the Mango and Allen smoothing adjustment.

Solution:

18. Continued

(a) Select age-to-age development factors for all periods excluding the tail factor. Justify your selections.

Adjusted Age-to-Age Development Factors Excluding the Large Claim

Accident Year	12-24	24-36	36-48	48-60	60-72	72-84	84-96
2013	3.191	1.675	1.352	1.197	1.122	1.091	1.063
2014	3.058	1.673	1.305	1.201	1.141	1.094	
2015	2.846	1.691	1.334	1.218	1.131		
2016	2.858	1.700	1.321	1.198			
2017	2.727	*1.726*	*1.332*				
2018	2.732	1.729					
2019	2.716						
All Years Avg.	2.876	1.699	1.329	1.204	1.131	1.092	1.063
Avg. excl. high&low	2.844	1.698	1.329				
Volume Wtd. Avg.	2.861	*1.699*	*1.329*	1.204	1.131	1.092	
5 Year Avg.	2.776	1.704					
3 Year Avg.	2.725	1.718	1.329	1.206			
Selected	2.725	1.718	1.329	1.206	1.131	1.093	1.063

Justification for selection: Selected 3 years average to recognize trend down the columns.

Notes: Adjusted factors for large claim:
AY2017, 24-36 = 1.726 = (1,082 – 150)/540
AY2017, 36-48 = 1.332 = (1,391 – 150)/(1,082 – 150)
Volume Wtd. Avg., 24-36: 1.699 = (866 + 875 + 876 + 923 + 1,082 + 968 – 150)/(517 + 523 + 518 + 543 + 540 + 560)
Volume Wtd. Avg., 36-48: 1.329 = (1,171 + 1,142 + 1,169 + 1,219 + 1,391 – 150)/(866 + 875 + 876 + 923 + 1,082 – 150)

(b) Derive a paid tail factor using Boor's algebraic method.

	(1)	(2)		(3) = (1)(2)	(4)	(5) = (4)/(3)
				Estimated Claims		
		Paid Development Factors			Ultimate Claims from Reported Development	
Accident Year	Actual Paid	72-84	84-96	96	Method	Implied Tail Factor
2013	1,824			1,824	1,975	1.083
2014	1,712		1.063	1,820	1,974	1.085
2015	1,610	1.093	1.063	1,870	2,032	1.087
					Selected:	1.085

18. Continued

(c) Calculate ultimate claims using the paid development method and the tail factor of 1.072.

Accident Year	(1) Actual Paid	(2) Paid Claims Excluding Large Claim	(3) Age-to-Ultimate Development Factors	(4) = (2)(3) Ultimate Claims
2013	1,824	1,824	1.072	1,955
2014	1,712	1,712	1.140	1,951
2015	1,610	1,610	1.245	2,004
2016	1,460	1,460	1.408	2,056
2017	1,391	*1,241*	1.698	*2,257*
2018	968	968	2.257	2,184
2019	573	573	3.877	2,222
2020	224	224	10.566	2,367
Total	9,762	9,612		16,997

e.g., *1,241* = 1,391 – 150
1.698 = 1.206×1.131×1.093×1.063×1.072

(d) Calculate the unpaid ULAE as of December 31, 2020 using the classical paid-to-paid method and a multiplier of 50%.

Case outstanding = 14,660 – 9,762 = 4,898
IBNR = 17,065 – 14,660 = 2,405
Unpaid ULAE = 0.08×2,405 + 0.8×0.5×4,898 = 388.

(e) Describe the Kittel refinement to the classical paid-to-paid method and the weakness it is designed to address.

Kittel method derives ULAE ratio by comparing paid ULAE to average of paid and reported claims (rather than paid to paid ratio used in Classical method).

Kittel's change addresses some of the distortion that can arise with increasing (changing) exposures because reported claims react quicker to exposure changes.

(f) Describe the Mango and Allen smoothing adjustment.

The Mango and Allen Smoothing Adjustment uses expected claim in place of actual claims.

19. Learning Objectives:

2. The candidate will demonstrate the ability to prepare claims and exposure data for general insurance actuarial work.

6. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

Learning Outcomes:

(2c) Calculate written, earned, in-force and unearned premiums for portfolios of policies with various policy terms and earnings patterns.
(2d) Adjust historical earned premiums to current rate levels.
(6c) Describe the purpose of base rates and rating factors and explain how they are used to determine an insured's premium.
(6j) Calculate indicated rates and indicated rate changes using the claim ratio and pure premium methods.

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 11, 12, 28, and 31.

Commentary on Question:

This question tests the candidate's understanding of the earning of exposures and adjusting premiums to current rate levels.

Solution:

(a) Calculate the calendar year 2019 total earned premiums at current rate levels using the extension of exposures method.

Commentary on Question:

Premiums need to be adjusted to current rate levels.

	(1)	(2)	(3)	(4)	(5)	(6)	(7) = (2)(4)(5)(6)
Policy	Territory	Territory Factor	Class	Class Factor	Base Rate	Term (years)	Earned Premium at Current Rate Level
1	3	0.94	1	1.18	805	1.000	892.91
2	2(1)	1.30	2	1.00	805	0.417	436.04
2	1(2)	1.00	2	1.00	805	0.583	469.58
3	2	1.30	1	1.18	805	1.000	1,234.87
Total							3,033.40

Notes: (1) Policy 2 is rated as territory 2 from January 1 through May 31
(2) Policy 2 is rated as territory 1 from June 1 through December 31

19. Continued

(b) Calculate the total in-force premiums as of July 1, 2019 using the July 1, 2018 rates.

Policy	(1) Territory	(2) Territory Factor	(3) Class	(4) Class Factor	(5) Base Rate	(6) = (2)(4)(5) Earned Premium at Current Rate Level
1	3	0.97	1	1.15	780	870.09
2	1	1.00	2	1.00	780	780.00
3	2	1.25	1	1.15	780	1,121.25
Total						2,771.34

(c) Calculate the total in-force premiums as of July 1, 2019 using the July 1, 2019 rates.

Policy	(1) Territory	(2) Territory Factor	(3) Class	(4) Class Factor	(5) Base Rate	(6) = (2)(4)(5) Earned Premium at Current Rate Level
1	3	0.95	1	1.12	795	845.88
2	1	1.00	2	1.00	795	795.00
3	2	1.22	1	1.12	795	1,086.29
Total						2,727.17

(d) Calculate the overall premium change for the July 1, 2019 rate changes.

Overall premium change = (Total Premium @ July 1, 2019 rates / (Total Premium @ July 1, 2018 Rates) – 1 = (part (c)) / (part (b)) – 1 = 2,727.17 / 2,771.34 – 1
= –1.59%

(e) Explain why the July 1, 2019 increase in the base rate is not equal to the overall premium change calculated in part (d).

The difference is due to the various rating factors and accounting for the different rating characteristics of each policy.

20. Learning Objectives:

6. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

Learning Outcomes:

(6d) Quantify different types of expenses required for ratemaking including expense trending procedures.

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 29.

Commentary on Question:

This question tests the candidate's understanding of expenses used for ratemaking.

Solution:

(a) Calculate the total variable expense ratio for each calendar year.

	(1)	(2)	(3)	(4) = (2) + (3)
	General Expenses		Commission and	
Calendar Year	Variable	As a % of Premiums	Premium Tax Expense Ratio	Total Variable Expense Ratio
2018	1,016,250	11.68%	15.91%	27.59%
2019	1,087,500	11.57%	15.83%	27.40%
2020	1,117,500	11.29%	15.88%	27.17%

Notes: (1) = 75%×(General Expenses)
(2) = (1) / (Direct Earned Premium)
(3) = (Total Commission Expenses and Premium Taxes) / (Direct Written Premium)

(b) Recommend the total variable expense ratio to use in ratemaking. Justify your recommendation.

Calendar Year	Variable General Expense Ratio	Commission and Premium Tax Expense Ratio	Total Variable Expense Ratio
2018	11.68%	15.91%	27.59%
2019	11.57%	15.83%	27.40%
2020	11.29%	15.88%	27.17%
Average	11.51%	15.87%	27.39%
Selection:	11.29%	15.87%	27.16%

20. Continued

Justification:

- Variable general expense ratio is decreasing, so recommend the latest year of 11.29% to reflect the decrease
- Commission and premium tax expense ratio is steady so recommend the average of all 3 years, or 15.87%
- Recommended total variable expense ratio = 11.29% + 15.87% = 27.16%

(c) Recommend the fixed expense per exposure to use in ratemaking. Justify your recommendation.

Calendar Year	(5) Fixed General Expense (000)	(6) Fixed General Expense Per Exposure
2018	338,750.0	10.42
2019	362,500.0	10.76
2020	372,500.0	10.61
Average		10.60

Notes: (5) = 25%×(General Expenses)
(6) = (5) / (Earned Exposures)

Recommended fixed general expense per exposure = 10.60 (no significant trend so average of all 3 years is reasonable)

Provision for new system = 1,200,000 / 37,000 / 5 = 6.49 (amortize over 5 years)

Recommended fixed expense per exposure to use in ratemaking = 10.60 + 6.49 = 17.09