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Session 33 TS, Further Research on SOA Experience Study Calculations

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2018 SOA Life & Annuity Symposium

HEZHONG (MARK) MA

Session 33: Further Research on SOA Experience Study Calculations

Monday, May 7th, 2018

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- Do alert SOA staff and/or legal counsel to any concerning discussions
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Experience Study - Basics

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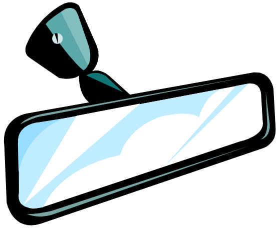
Experience Studies - Overview



- Study of Historical Experience
  - Non-parametric, widely-accepted actuarial practices with numerous ASOPs, practice notes.
- Life Year, Policy Year & Calendar Year
  - Risks – attained ages, select factors, and improvements
  - Methods – Policy year versus calendar year

Experience Studies Formulas

- Exposure = Business at Risk of Lapsing or Claiming a Benefit
- Expected = Portion of Business Expected to Lapse or Claim a Benefit

- Claim = Actual Amount Paid in Benefits

- q_x = probability of a claim occurring for a person aged (x) in the next year
- w_x = probability of a lapse occurring for a person aged (x) in the next year
- Expected Claim Count = Exposure Count * qx

Amount vs. Count

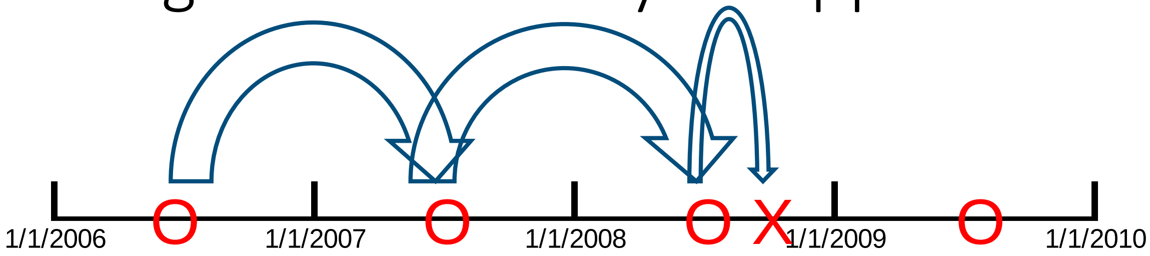
- Count is usually smoother but Amount is the "bottom line" metric
- Amount is obviously more volatile, especially when the cell of interest is small
- A higher A/E by Count vs A/E by Amount could indicate better than anticipated mortality at the higher policy sizes/face amounts
- If expected basis varies by face bands, it could mask the face amounts effect
- Face amount can correlate to other factors.

Fractional exposure vs. fractional rates

Origin of Partial Year problem

Extension could be material for high termination rate situations.

The danger of calendar year approach on lapse



Define a Study

- Policy Year Study Example
  - Start Point – First anniversary within the study period
  - End Point – The first of:
    - Last anniversary within study period
    - Termination date within study period
  - Appropriate when terminations are skewed within a duration
- Calendar Year Study Example
  - Fixed start and stop point for all exposure calculations
  - Suitable when claims or terminations are not skewed
  - Can provide more current information and increase credibility

Policy Year Study

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Experience Study Data Setup

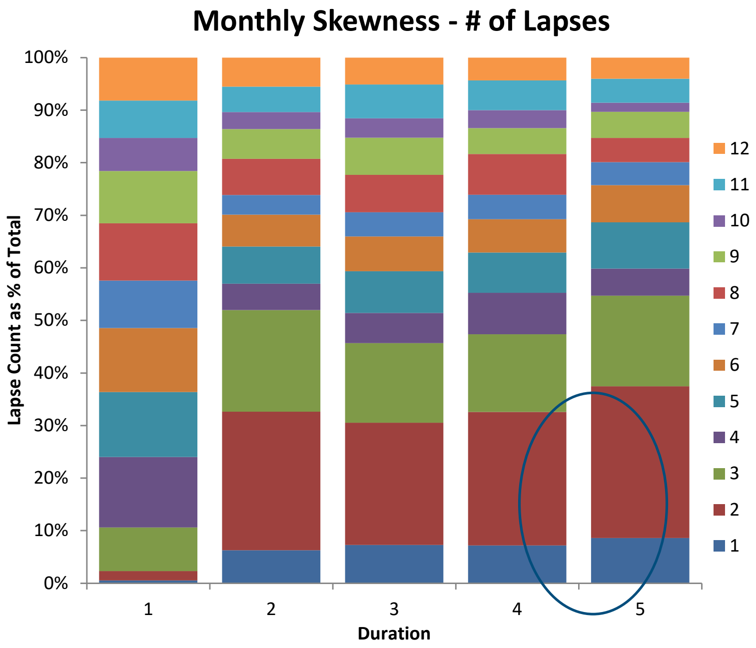
Lapse Dates and Lapse Imputing

- Lapses
  - Skewed towards premium payment date
  - Lapse date can be affected by grace period
    - Paid-to Date (Effective Date) to be used in studies

- When lapse dates not available
  - Impute Lapses – Estimate date of lapse
  - If annual pay business
    - Lapse date = anniversary date, Does not start a new duration
  - Other mode of payments
    - Consider mid-period lapses

Data Profiling Example:

Lapse Skewness for One Product

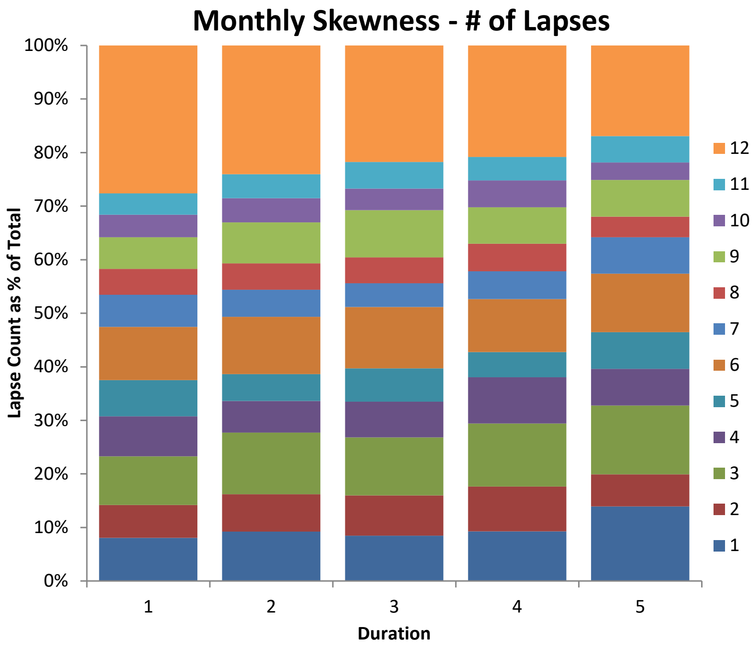


Problem:

- Appear to be missing month 1 & 2 lapses
- Lapses shifted to the beginning of policy years 2+

Data Profiling Example:

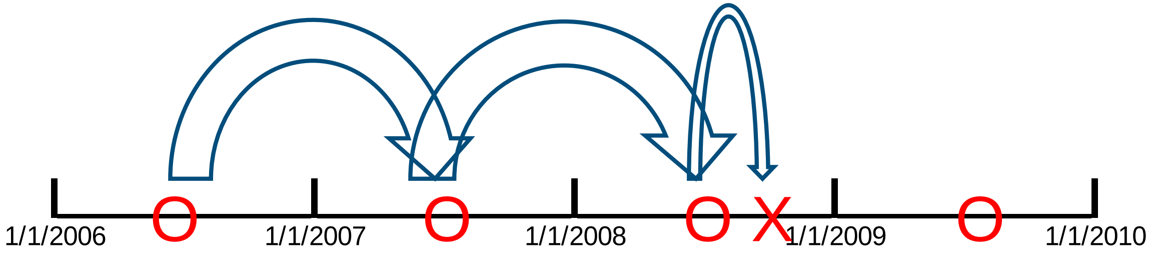
Lapse Skewness for One Product



Solution:

- Terminations included the grace period
- Shift lapses back – resulted in a normal skewness pattern

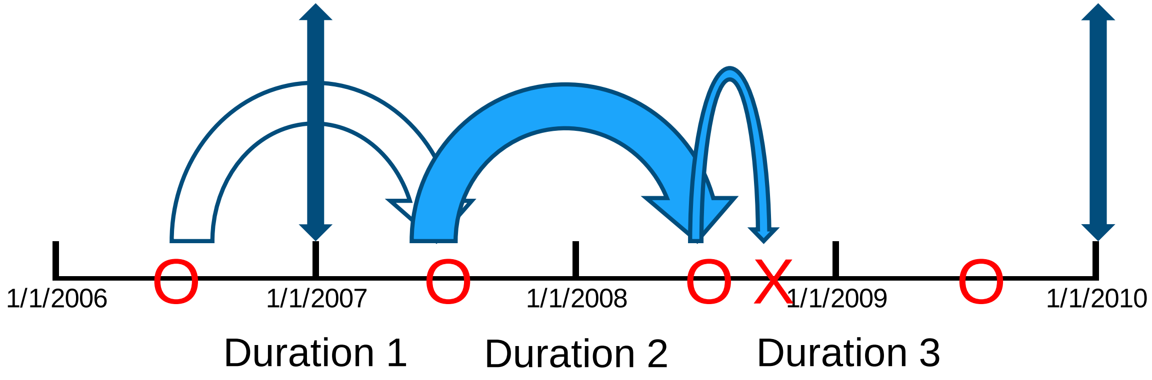
Example



- Issue Date: 6/30/2006
- Termination Date: 9/30/2008
- Face Amount: $100,000 (level)
- $q_x = 0.1$, $q_{x+1} = 0.2$, $q_{x+2} = 0.3$

Lapse Study Example: Death

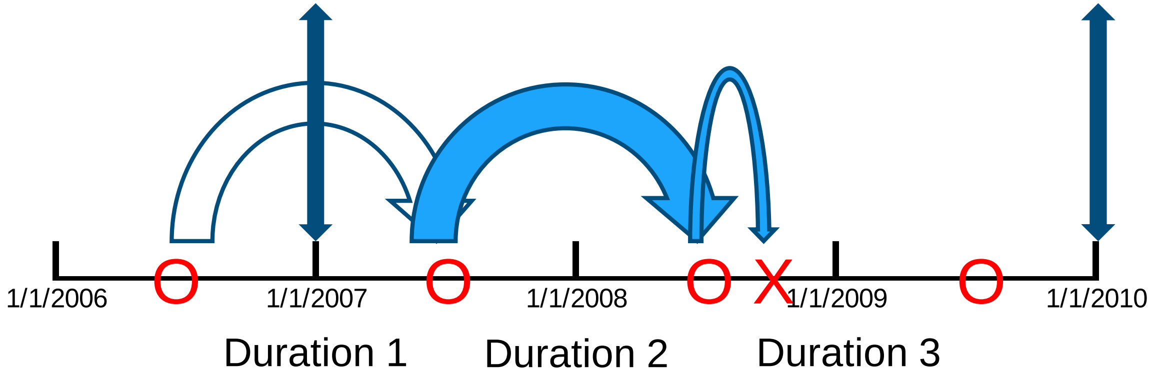
- Issue Date: 6/30/2006
- Termination Date: 9/30/2008
- Face Amount: $100,000



- 2007 – 2009 Lapse Study
  - Lapse study should be an anniversary year study
    - Only include anniversary years 2007-2008 and 2008-2009
  - Exposure begins at the first anniversary inside the study period
  - A death's exposure must end at the termination date inside study period

Lapse Study Example: Death

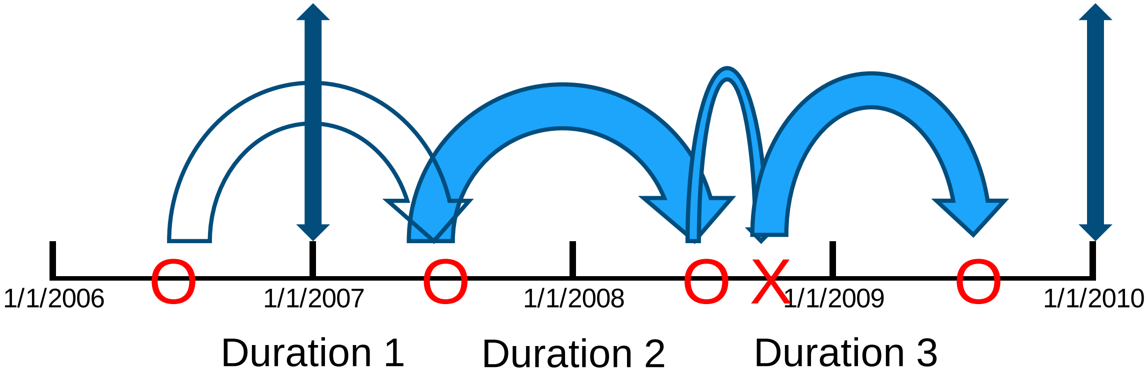
- Issue Date: 6/30/2006
- Termination Date: 9/30/2008
- Face Amount: $100,000



- 2007 – 2009 Lapse Study
  - Exposure Count:
  - 2007-08 Duration 2 = 1
  - 2008-09 Duration 3 = ¼
  - Exposure Amount:
  - 2007-08 Dur 2 = 100,000
  - 2008-09 Dur 3 = 25,000

Lapse Study Example: Lapse

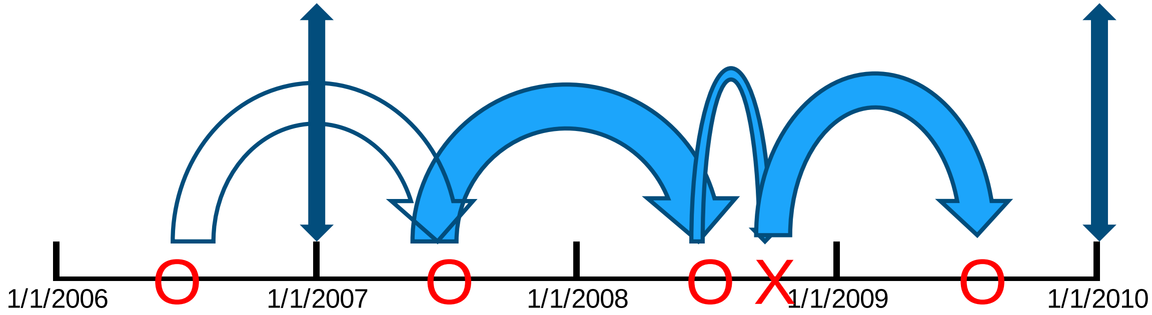
- Issue Date: 6/30/2006
- Termination Date: 9/30/2008
- Face Amount: $100,000



- 2007 – 2009 Lapse Study
  - Exposure begins at the later of:
    - Issue Date
    - Study Start Date
  - A lapse must end exposure at the next anniversary after the lapse

Lapse Study Example: Lapse

- Issue Date: 6/30/2006
- Termination Date: 9/30/2008
- Face Amount: $100,000



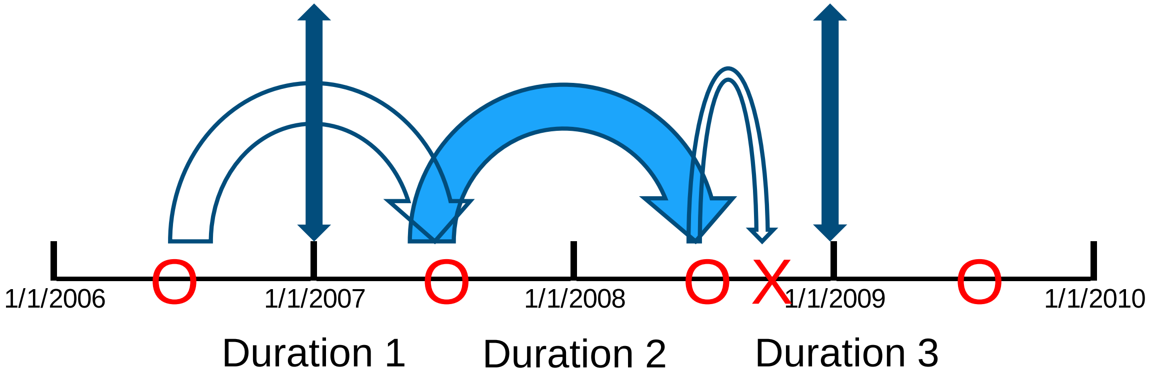
Duration 1 Duration 2 Duration 3

- 2007 – 2009 Lapse Study
  - Exposure Count:
  - 2007-08 Duration 2 = 1
  - 2008-09 Duration 3 = 1

- Policy Contributes Lapse Count = 1, Lapse Amount = 100,000
  - (in duration 3 of study period 2008-09)

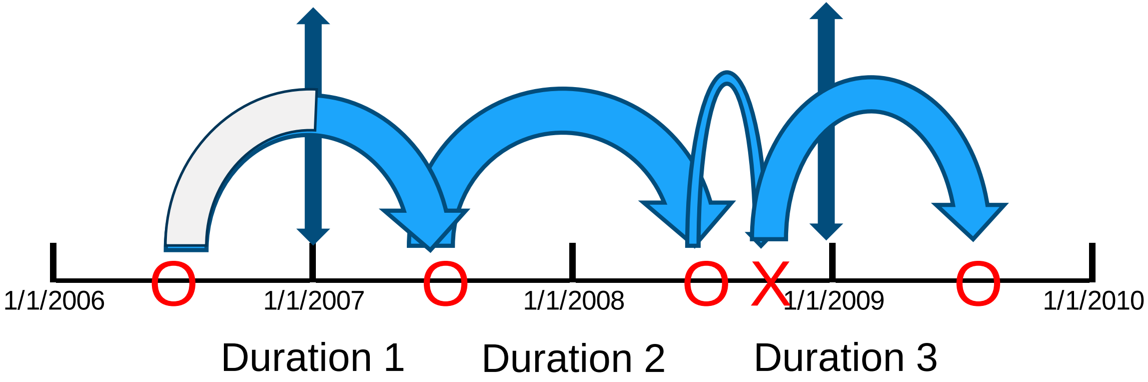
Lapse Study Example: Lapse

- Issue Date: 6/30/2006
- Termination Date: 9/30/2008
- Face Amount: $100,000

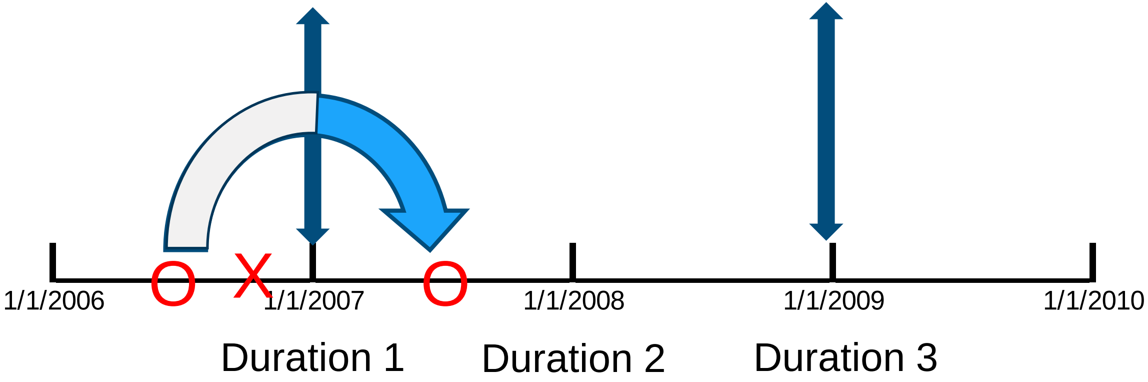


- 2007 – 2008 Lapse Study
  - Exposure Count:
  - 2007-08 Duration 2 = 1
- No exposure for 2008-2009
  - It is outside the study period (Policy Year Study)
- Do not count actual lapse in study
  - It is outside the study period

Annual/Traditional Exposure Method



- Issue Date: 6/30/2006
- Termination Date: 9/30/2008
- Face Amount: $100,000
- Termination Cause: Studied Event
- Study Period:
  1/1/2007 to 12/31/2008



- Issue Date: 6/30/2003
- Termination Date: 9/30/2006
- Face Amount: $100,000
- Termination Cause: Studied Event
- Study Period:
  1/1/2007 to 12/31/2008

Calendar Year Study

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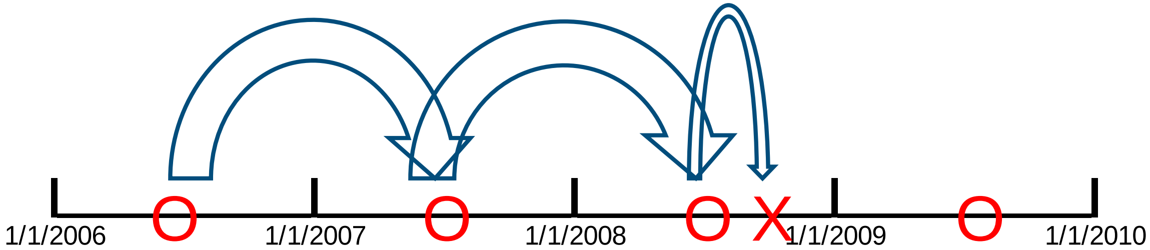
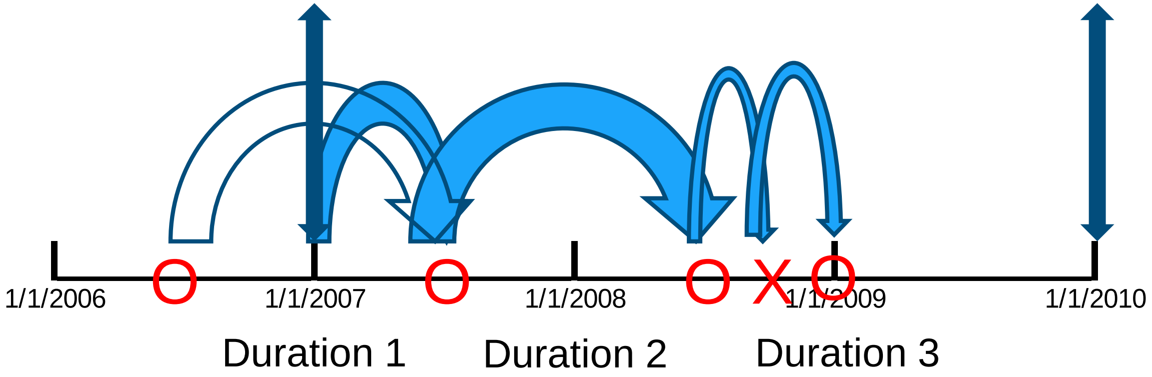
Example



- Issue Date: 6/30/2006
- Termination Date: 9/30/2008
- Face Amount: $100,000 (level)

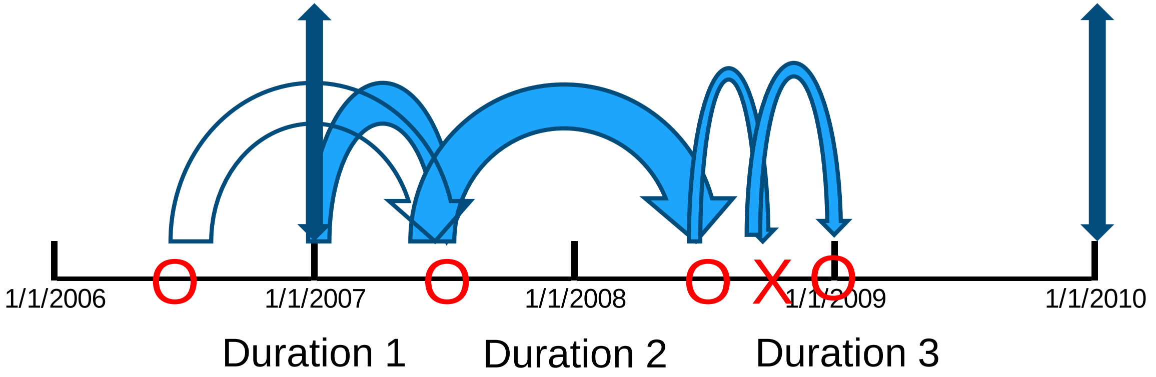
Mortality Study Example: Death



- Issue Date: 6/30/2006
- Termination Date: 9/30/2008
- Face Amount: $100,000

- 2007 – 2009 Mortality Study
  - Exposure begins at the later of:
    - Issue Date
    - Study Start Date
  - Initial exposure method: A death extends exposure to the end of the calendar year of death; for a partial calendar year study, to the end of the study period if it is between the date of death and the end of the calendar year of death.

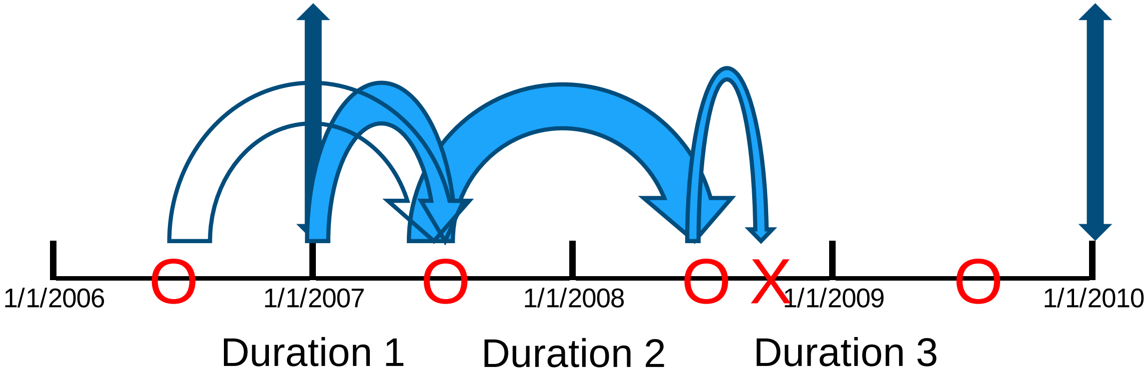
Mortality Study Example: Death



- Issue Date: 6/30/2006
- Termination Date: 9/30/2008
- Face Amount: $100,000

- 2007 – 2009 Mortality Study
  - Exposure Count:
  - 2007 Duration 1 = ½
  - 2007 Duration 2 = ½
  - 2008 Duration 2 = ½
  - 2008 Duration 3 = ½
  - Exposure Amount:
  - 2007 Duration 1 = 50,000
  - 2007 Duration 2 = 50,000
  - 2008 Duration 2 = 50,000
  - 2008 Duration 3 = 50,000

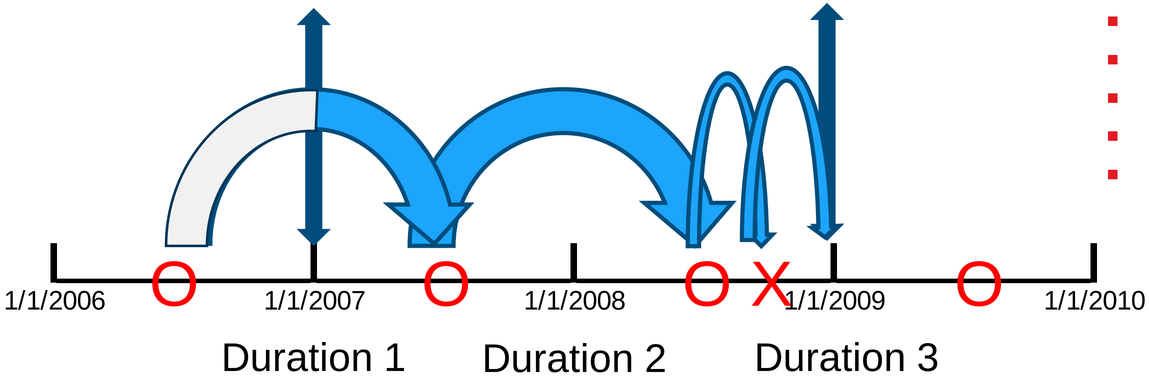
Mortality Study Example: Lapse



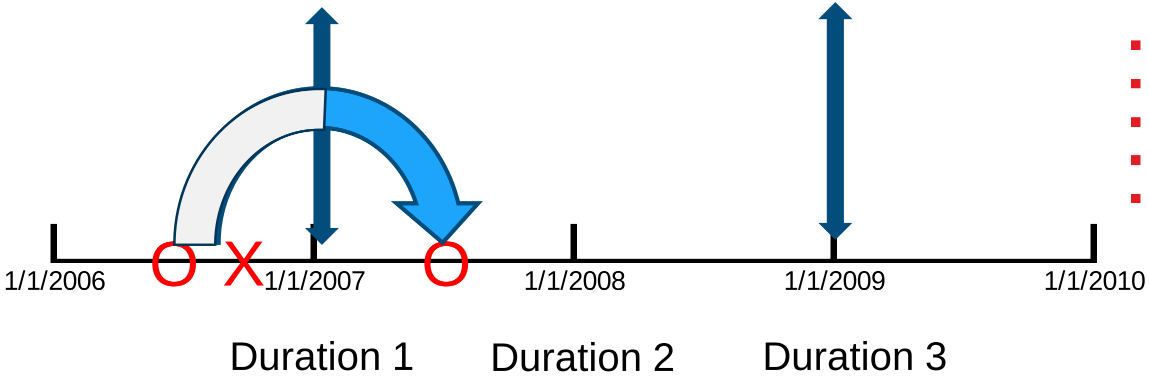
- Issue Date: 6/30/2006
- Termination Date: 9/30/2008
- Face Amount: $100,000

- 2007 – 2009 Mortality Study
  - Exposure begins at the later of:
    - Issue Date
    - Study Start Date
  - A lapse must end exposure at the date of termination.
  - In force policy ends exposure at the study end date.

Distributed Exposure Method



- Issue Date: 6/30/2006
- Termination Date: 9/30/2008
- Face Amount: $100,000
- Termination Cause: Studied Event
- Study Period:
  1/1/2007 to 12/31/2008



- Issue Date: 6/30/2003
- Termination Date: 9/30/2006
- Face Amount: $100,000
- Termination Cause: Studied Event
- Study Period:
  1/1/2007 to 12/31/2008

Add exposure but not claim.

Extend whom to when, for what?

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Uniform Distribution of Deaths (UDD)

${}_tq_x = t * q_x$

- The mortality rate for the fraction of the year from time 0 to t is proportional to the mortality rate for the full year. If you have 120 deaths in a year, UDD means 10 deaths a month.
- If we use the distributed exposure method (Ch5.), implicitly, the deaths are assumed uniformly distributed
- With decreasing in force, (due to terminations), this means the force of decrement is increasing
- Number of Lives is the Linear interpolation between two ages
- ${}_{1-t}q_{x+t} = (1 - t) * q_x / (1 - t * q_x)$

Balducci/Hyperbolic

$_{1-t}q_{x+t} = (1 - t) * q_x$

- The mortality rate from time t to the end of the year is proportional to the mortality rate for the full year

- If we use the annual exposure method, implicitly, the deaths are assumed to follow Balducci hypothesis.

- $_tq_x = t * q_x / [1 – (1 – t) * q_x]$

- Opposite to UDD, the fractional rate decreases. The partial year mismatch could be big when both lapse and mortality are high

- $_tp_x$ is hyperbolic curve.

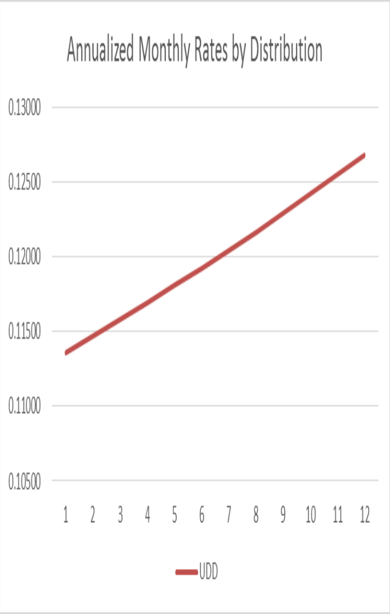
Constant Force

$_tq_x = 1 - (1 - q_x)^t$

- The mortality rate is constant over all of the fractional periods
- Number of Lives is the Exponential interpolation between two ages

Uniform Distribution of Deaths (UDD)

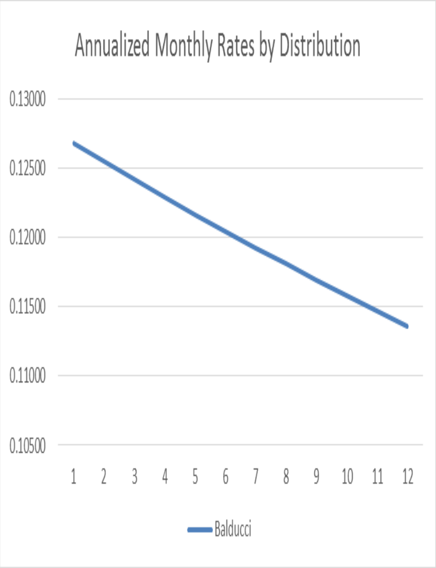
${}_tq_x = t * q_x$



9.1.2 UDD/Linear Interpolations					
	beginning number of lives	number of deaths	monthly exposure	monthly mortality rate	annualized equivalent
	(1)	(2)	(3)	(4)	(5)
	1(t-1)-2(t-1)	1(0)/12	1(t)	2(t)/3(t)	1-(1-4(t))^12
0	1,000.0	10.0	1,000.0	0.01000	0.11362
1	990.0	10.0	990.0	0.01010	0.11470
2	980.0	10.0	980.0	0.01020	0.11581
3	970.0	10.0	970.0	0.01031	0.11693
4	960.0	10.0	960.0	0.01042	0.11808
5	950.0	10.0	950.0	0.01053	0.11925
6	940.0	10.0	940.0	0.01064	0.12045
7	930.0	10.0	930.0	0.01075	0.12167
8	920.0	10.0	920.0	0.01087	0.12291
9	910.0	10.0	910.0	0.01099	0.12418
10	900.0	10.0	900.0	0.01111	0.12548
11	890.0	10.0	890.0	0.01124	0.12680
12					
Total		120.0	11,340.0	0.010582	0.11985

Balducci

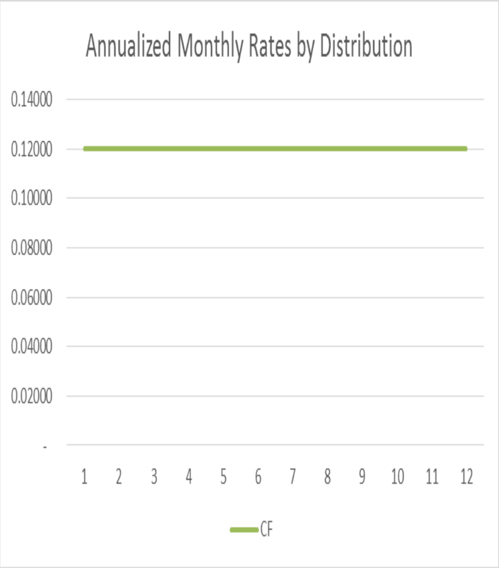
$${}_{1-t}q_{x+t} = (1 - t) * q_x$$



9.1.1 Balducci Hypothesis (Hyperbolic/Harmonic Interpolations)						
	beginning number of lives	number of deaths	monthly exposure	cumulative mortality rate from the start of age x	monthly mortality rate	annualized equivalent
	(1)	(2)	(3)		(4)	(5)
	1(t-1)-2(t-1)	1(t)*4(t)	1(t)	Month/12*annual_Qx/(1-((1-month/12)*annual_Qx))		1-(1-4(t))^12
0	1,000.0	11.2	1,000.0	0.01124	0.01124	0.12680
1	988.8	11.0	988.8	0.02222	0.01111	0.12548
2	977.8	10.7	977.8	0.03297	0.01099	0.12418
3	967.0	10.5	967.0	0.04348	0.01087	0.12291
4	956.5	10.3	956.5	0.05376	0.01075	0.12167
5	946.2	10.1	946.2	0.06383	0.01064	0.12045
6	936.2	9.9	936.2	0.07368	0.01053	0.11925
7	926.3	9.6	926.3	0.08333	0.01042	0.11808
8	916.7	9.5	916.7	0.09278	0.01031	0.11693
9	907.2	9.3	907.2	0.10204	0.01020	0.11581
10	898.0	9.1	898.0	0.11111	0.01010	0.11470
11	888.9	8.9	888.9	0.12000	0.01000	0.11362
12						
Total		120.0	11,309.6		0.01061	0.12015

Constant Force

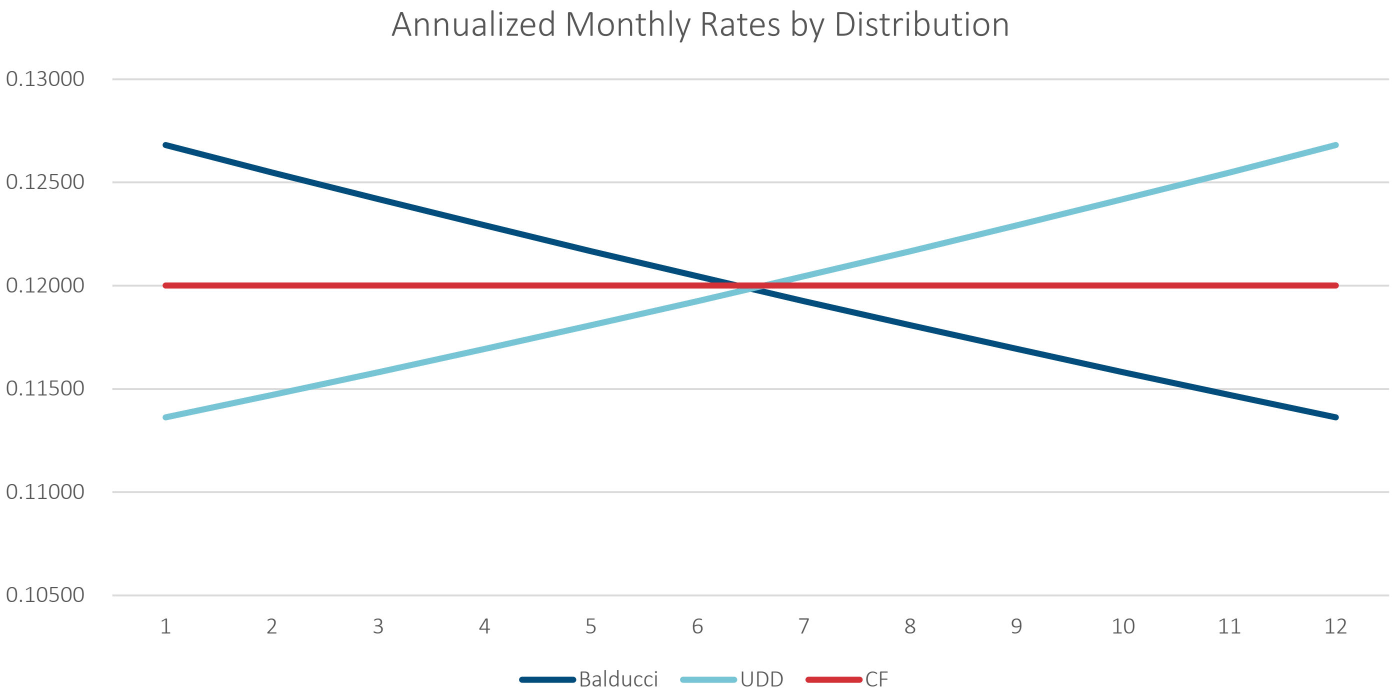
$_tq_x = 1 - (1 - q_x)^t$



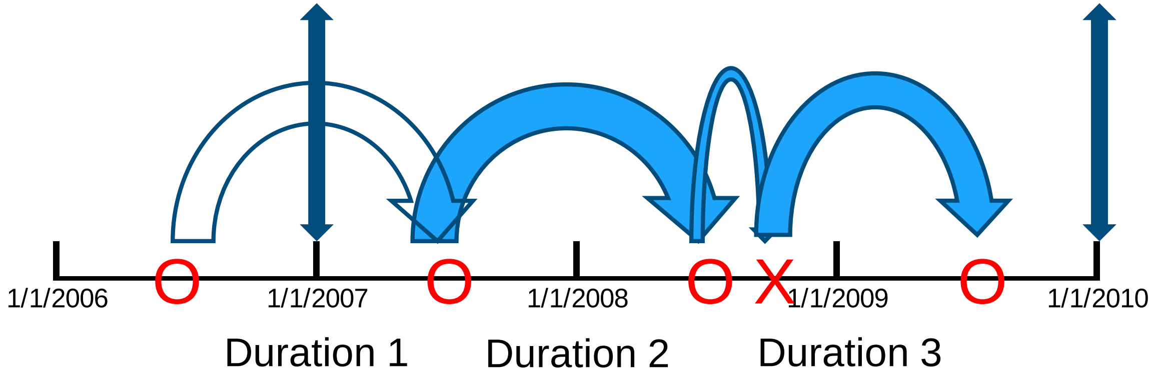
9.1.3 Constant Force/Exponential Interpolation					
	beginning number of lives	number of deaths	monthly exposure	monthly mortality rate	annualized equivalent
	(1)	(2)	(3)	(4)	(5)
		1(t)*4(t)		1-(1-annual_Qx)^(1/12)	
0	1,000.0	10.6	1,000.0	0.01060	0.12000
1	989.4	10.5	989.4	0.01060	0.12000
2	978.9	10.4	978.9	0.01060	0.12000
3	968.5	10.3	968.5	0.01060	0.12000
4	958.3	10.2	958.3	0.01060	0.12000
5	948.1	10.0	948.1	0.01060	0.12000
6	938.1	9.9	938.1	0.01060	0.12000
7	928.1	9.8	928.1	0.01060	0.12000
8	918.3	9.7	918.3	0.01060	0.12000
9	908.6	9.6	908.6	0.01060	0.12000
10	899.0	9.5	899.0	0.01060	0.12000
11	889.4	9.4	889.4	0.01060	0.12000
12					
Total		120.0	11,324.8	0.010596	0.12000

Make a choice

Spreadsheet examples

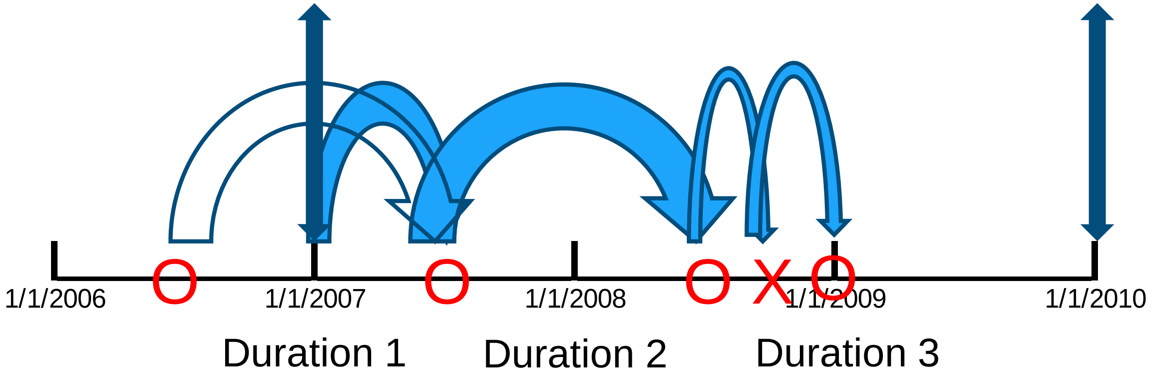


Policy Year Study Example: Studied Event



- Extending to the coming anniversary -> Balducci Hypothesis
- The rate for the Second Partial Year is proportional to annual rate.
  - [x+t,x+1): ${}_{1-t}q_{x+t} = (1-t)q_x$.
- The rate is decreasing over the year.

Calendar Year Study Example: Studied Event



Extending to the end of Calendar Year -> UDD

The rate for the First Partial Year is proportional to the annual rate.

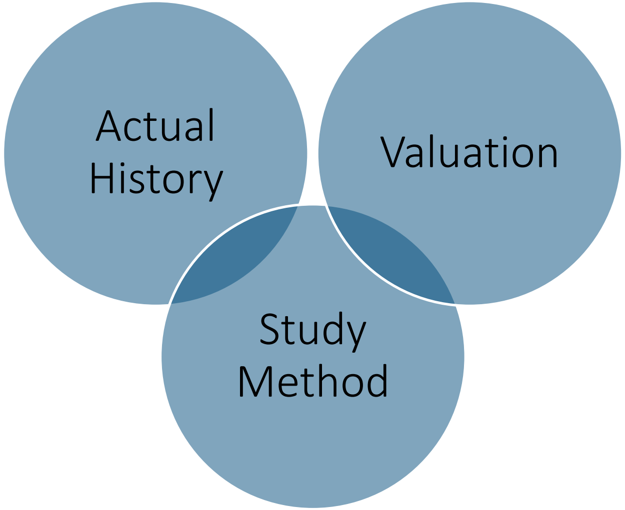
[x,x+t): ${}_t q_x = t q_x$

The rate is increasing over year.

The deaths are distributed uniformly over the year.

The Right Answer is,

It depends,



- **Actual History**

Distribution of decrements

Multiple decrements

- **Study Method**

Define study

Extend exposure

Assumption Setting and Projection

Convert into Monthly rates

Skewness Factors

Reproduce History

Suppose history is constant force

	BOM Surv	# Death	q	
Jan	100,000	1,000	1.00%	99,500
Feb	99,000	990	1.00%	98,505
Mar	98,010	980	1.00%	97,520
Apr	97,030	970	1.00%	96,545
May	96,060	961	1.00%	95,580
Jun	95,099	951	1.00%	94,624
Jul	94,148	941	1.00%	93,678
Aug	93,207	932	1.00%	92,741
Sep	92,275	923	1.00%	91,814
Oct	91,352	914	1.00%	90,895
Nov	90,438	904	1.00%	89,986
Dec	89,534	895	1.00%	89,087
Jan	88,639			

- **Annual**

P =88.64% Q=11.36%

- **Extend Exposure to the end of calendar year**

exposure	100,000
claims	11,361
Annual Qx	11.36%
Month Qx UDD	0.95%
Month Qx CF	1.0000%

- **If we do not extend exposure, and death occurs in mid month**

exposure	94,679
claims	11,361
Annual Qx	12.00%
Month Qx UDD	1.0000%
Month Qx CF	1.06%

Experience Study Process – Best Practice

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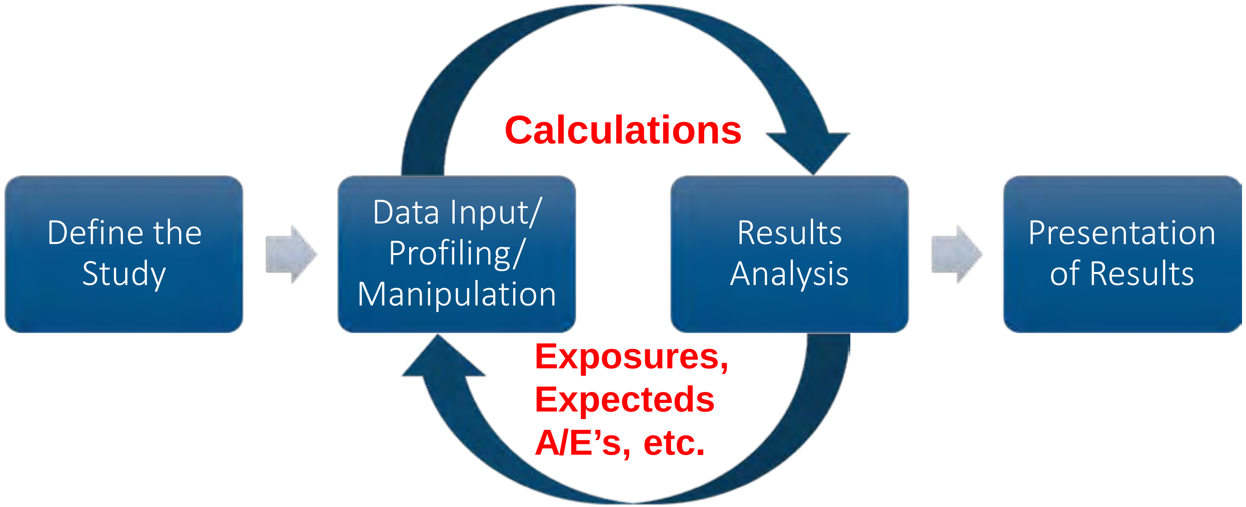
The Experience Study Process



Experience Study Process

- Process is a modular process
  - Data Input/Profiling/Manipulation
  - Calculations/Assumptions
  - Results Analysis
- Each piece of the process:
  - Is Stand-alone
  - Follows “Best Practices”
  - Is Easily Upgradable
- Sign-off of an experience study should include the entire process
  - A reviewer should follow the study from data input to results

Experience Study – Data Input/Profiling/Manipulation

- Process should exist to move data from an “admin” system to the experience study system
- Process should “Profile” the raw data
  - Provide metrics on the data (distributions by counts/amounts, estimated rates, etc.)
  - Metrics can be analyzed/presented/reviewed
- Process should allow for custom “Manipulation”
  - Mappings and manipulations should be saved for repeatability
  - Mapping process can be used for documentation for auditability

- **Generally the most neglected but most time consuming piece of the experience study process**

Challenges to Reinsurers and Industry Studies

- Try not to alter source data
- Multiple sources of truth
- Different admin practices (grace period? Policy status?)
- Exactly when did underwriting criteria change?
- History is lost
- Sometimes, data is simply unavailable

Consideration when analyzing experience

- Nuance
  - Biases in data or sampling procedures
  - Correlated or confounding predictor variables
  - Careful study design
  - Reporting lags / completion factors
  - Natural volatility, seasonality and other temporary “blips”
  - Temporal or cohort trends
  - External factors
  - Biometric drivers
  - Socio-economic influences
  - Behavioral dynamics
  - Changing mix of business
  - Lapse skewness/timing

- Infrastructure
  - Data quality
  - High-performance computing capabilities
  - Multivariate mapping dimensions
  - Advanced Analytics

- Communication
  - Effective presentation methods
  - Visual displays of quantitative data

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2018 SOA Life & Annuity Symposium

JOHN MCGARRY

Session 33: Further Research on SOA Experience Study Calculations

Saturday, May 7th, 2018

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Experience Studies: Annual Rate Errors

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SOA Experience Study Papers

- SOA Experience Study Calculations
  - By David B. Atkinson & John K. McGarry, Oct. 2016.
- SOA Experience Study Rate Errors
  - By John K. McGarry (Ed. David B. Atkinson), Nov. 2017.
- Available at:
  - www.soa.org/tables-calcs-tools/experience-study-tool/
- SOA Resources:
  - Cynthia McDonald, Korrel Rosenberg
  - Project Oversight Group (Calculations paper)

SOA Experience Study Rate Errors

- §2) Reviews the methods, formulae and relationships for full ages in a policy year mortality study, including fractional methods.
- §3) Develops the formula for rates and errors arising for partial ages with respect to the fractional method.
- §4) Investigates how exposures, rates and errors accumulate in a study.
  - Develops a simple cohort model for accumulating errors in a study.
- §5) Develops a model assuming force increases linearly over the year of age.
- §6) Tests the study methods using the models with VBT2015 select and ultimate rates.
  - Results not tested using actual experience study data.

Calendar-Year Studies

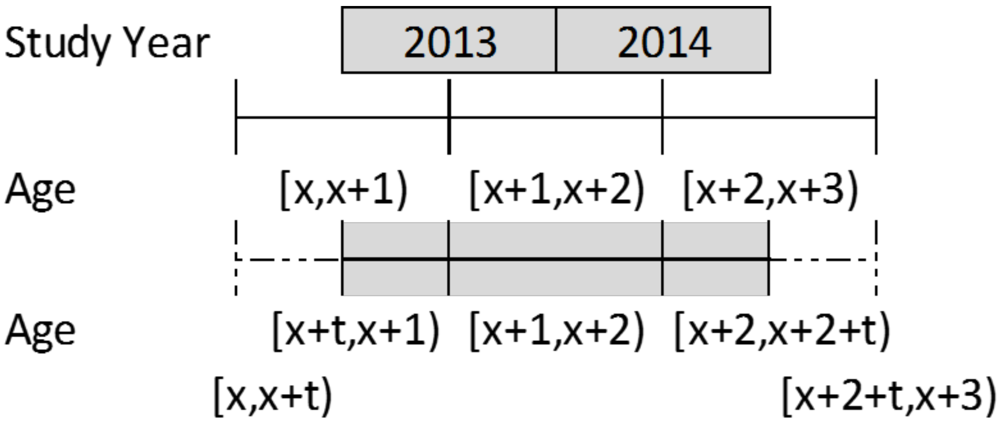
- Calendar Year studies tie in with accumulating year-end extracts.
- Calendar Year studies use all available data while Policy Year studies discard partial years at the start and end of the study.
- Calendar Year studies give true annual trends, while annual Policy Year trends are blended over 2 years.
- One Year Calendar Year studies tie experience to financial reporting, particularly for Group/YRT business.

Industry Study Methods

- Rate Year Traditional Method
  - Plan Year Study Period and Ages, e.g.
    - SOA RPEC Pension Mortality
  - Policy Year Study Period and Ages, e.g.
    - SOA Individual Life Mortality 2002-2009, VBT2015.
    - LIMRA Individual Life Persistency 2007-2009.
- Calendar Year Traditional Method
  - Calendar Year Study Period and Policy Year Ages, e.g.
    - SOA Individual Life Mortality 2009-2013.
- Force or Distributed method not found. (Reinsurers mainly)

Calendar-Year Studies: Partial Ages

- At the start and end of a calendar-year study, ages and study years intersect to give partial ages.
- A 2 year study period, 2013-14, has one complete policy year, starting in 2013, and two partial years at the start of 2013 and the end of 2013.



Exposure By Policy And Study Year

- 2 Year Study Period, 2013-14, Lives born 1959-62.
- Policy Years labeled by calendar year at the start of the policy year.
- Exposure split between the policy years at the start and end of the year.
- Pivots allow both Policy Year and Calendar Year study periods.

Exposure

Pol Yr	2012	2013		2014	Total
StdyYr	2013	2013	2014	2014	
50	996				996
51	977	1,126	961		3,065
52	882	1,115	935	1,088	4,020
53	814	1,002	853	1,073	3,742
54		937	781	973	2,690
55				895	895
Total	**3,669**	**4,180**	**3,530**	**4,029**	**15,408**

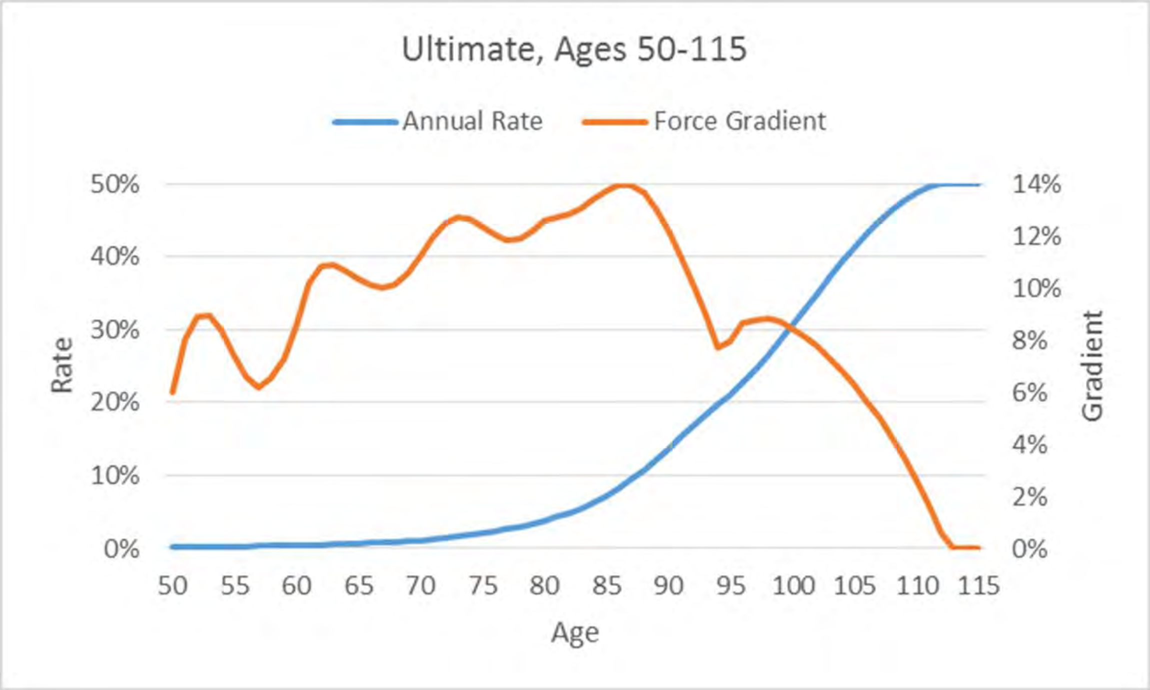
Partial Age Assumptions

- For partial ages (or years), the study methods assume decrements are proportional to time spent in the year, giving an implicit distribution of decrements.
- The difference between the implicit and actual distributions may distort the rates calculated in the study.
- For small rates or roughly uniform distributions, these distortions will not be material.

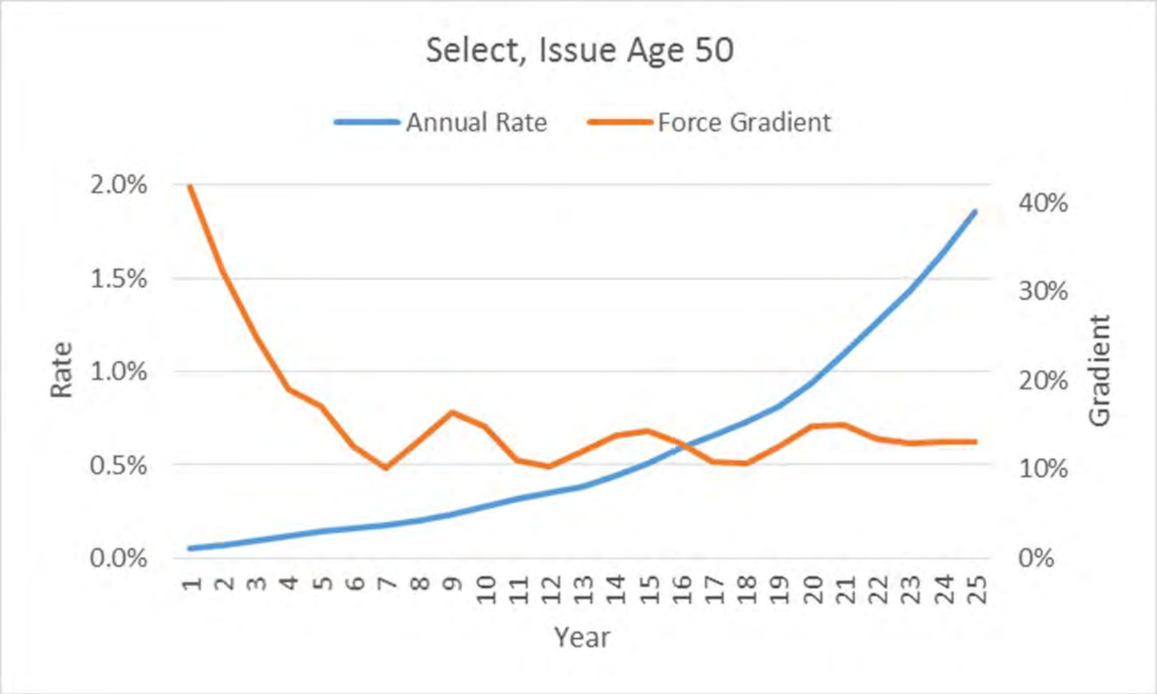
Actual Mortality Distributions

- As mortality is continuous, the distribution of deaths is determined by the increase in the force of mortality over the year.
- The increase in force for a given age is derived from the rates for the prior and following ages.
- The relative increase in force, i.e. the increase in force divided by the average force, or "gradient", Δ_x, gives the distribution independent of size of the rates across the age range.
- Actual experience assumed to match an industry table: VBT 2015 M NS ANB

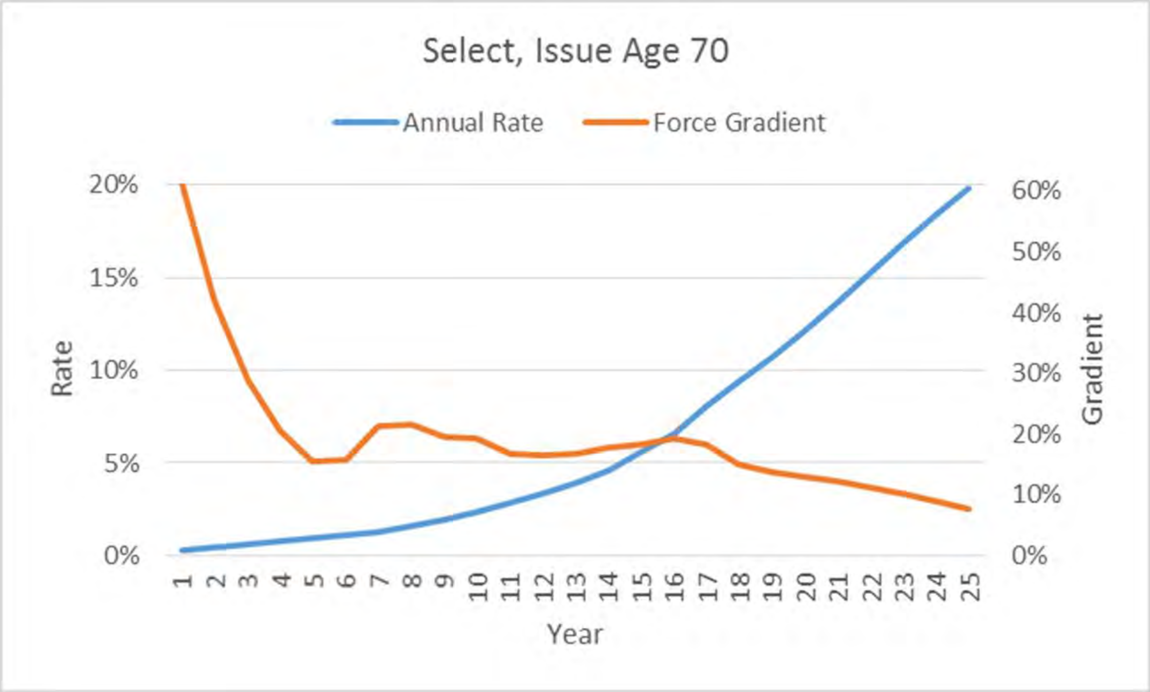
% Force Increase In Each Year



% Force Increase In Each Year



% Force Increase In Each Year

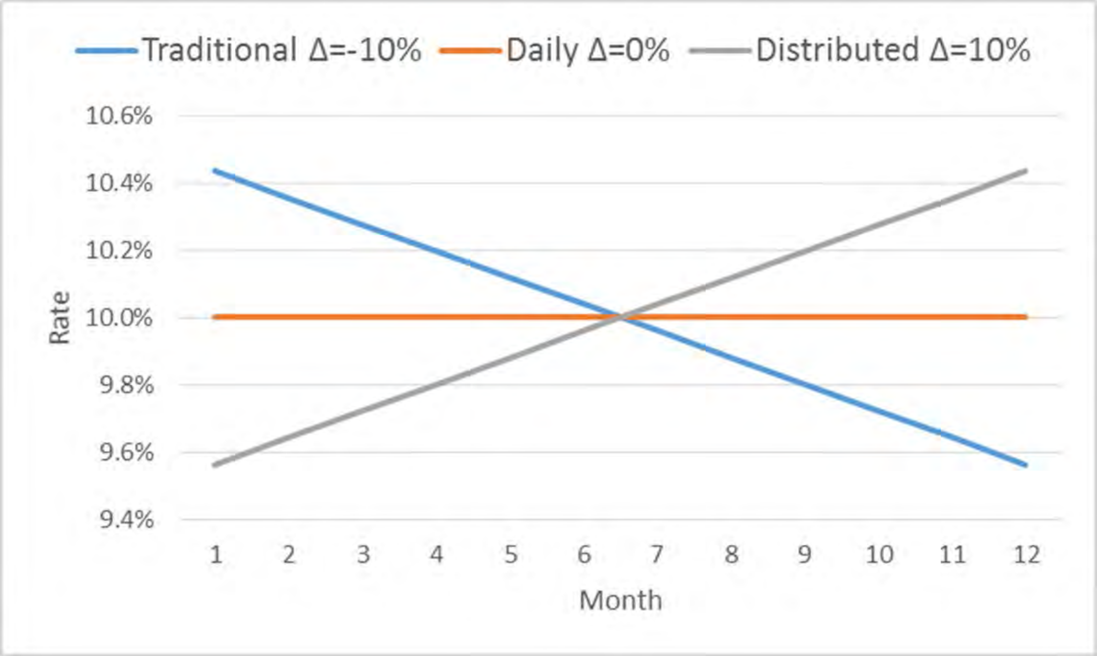


Study Methods

- Traditional Method
  - Annual Rate directly calculated from annual exposure that is the adjusted number of lives at the start of the age. For partial ages, deaths assigned exposure to the end of the age and the implied rate is decreasing over the year – Balducci hypothesis (BH).
- Daily Method
  - Annual Force calculated from annual exposure that is the amount of time spent in the year, from which the annual rate is calculated. For partial ages, the implied force is constant over the year (CF).
- Distributed Method
  - As Traditional, except for partial ages, deaths are assigned exposure to the end of the partial age only, and the implied rate is increasing over the year with deaths uniformly distributed (UD).

Partial Age Method Assumptions

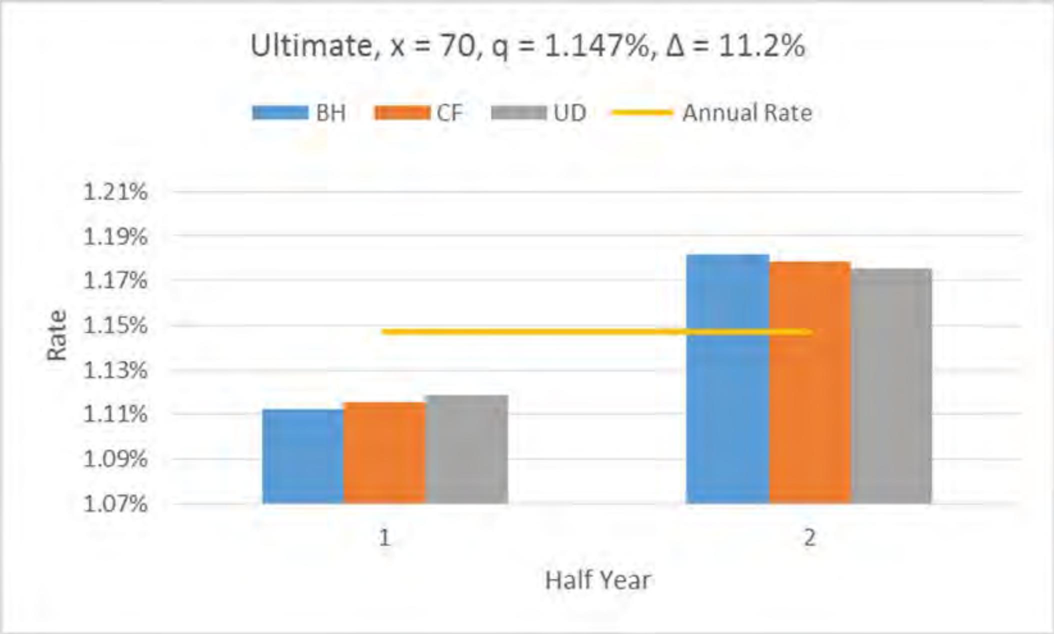
- Linear Force estimate
- Traditional: $\Delta_x \approx -q_x$
- Daily: $\Delta_x = 0$
- Distributed: $\Delta_x \approx +q_x$



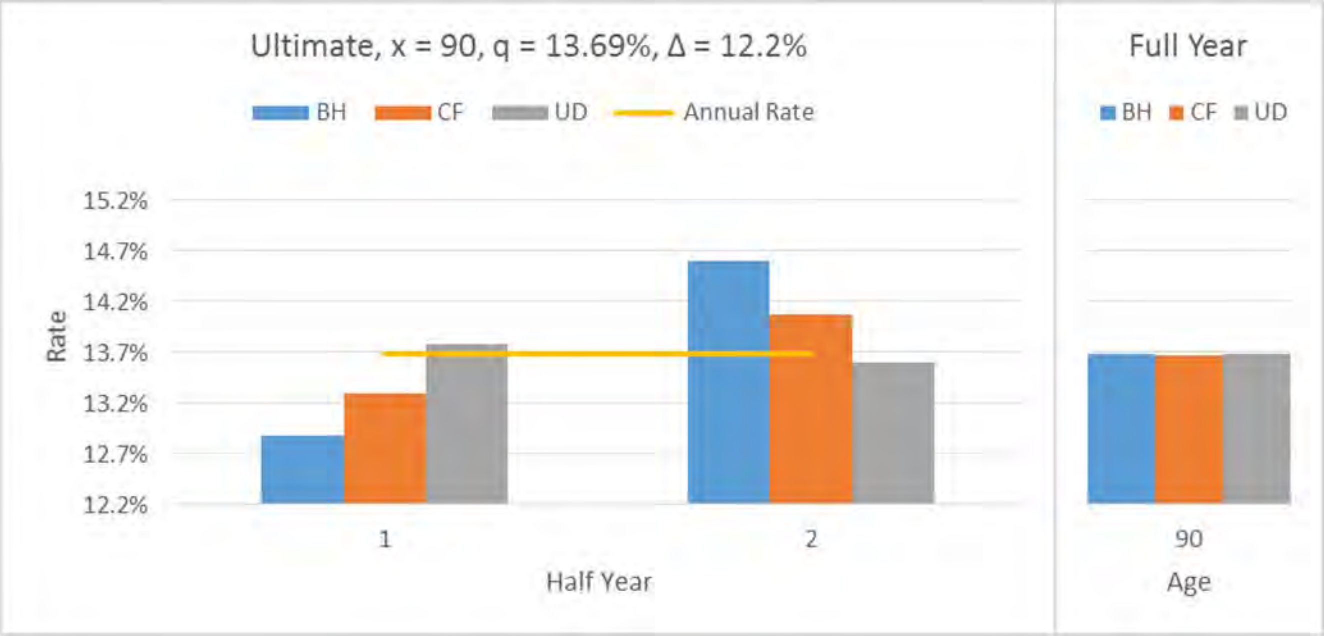
Half Year Rates And Errors

- If the age anniversaries are uniformly distributed over the year, the rates for partial ages that arise in a study can be estimated using half-year ages.
- The errors at the start of the study will be equal in size and opposite in sign to the errors at the end of the study.
- For all ages, the percentage errors in the rates using VBT 2015 M NS ANB.

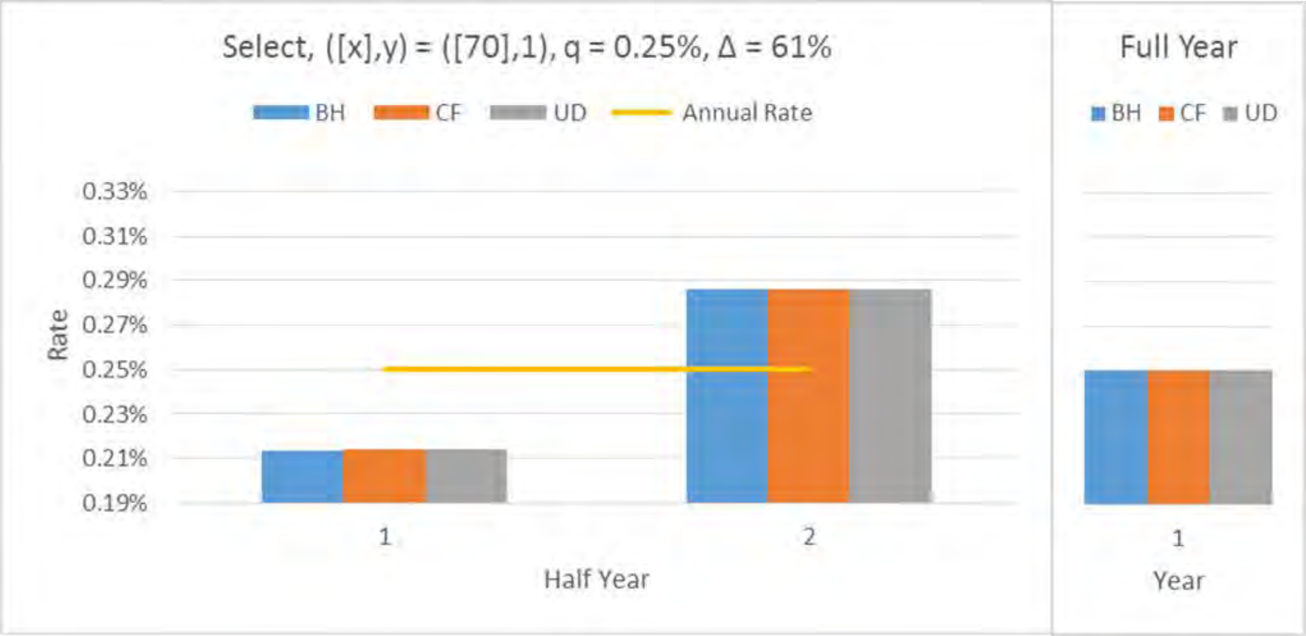
Half Year Rates



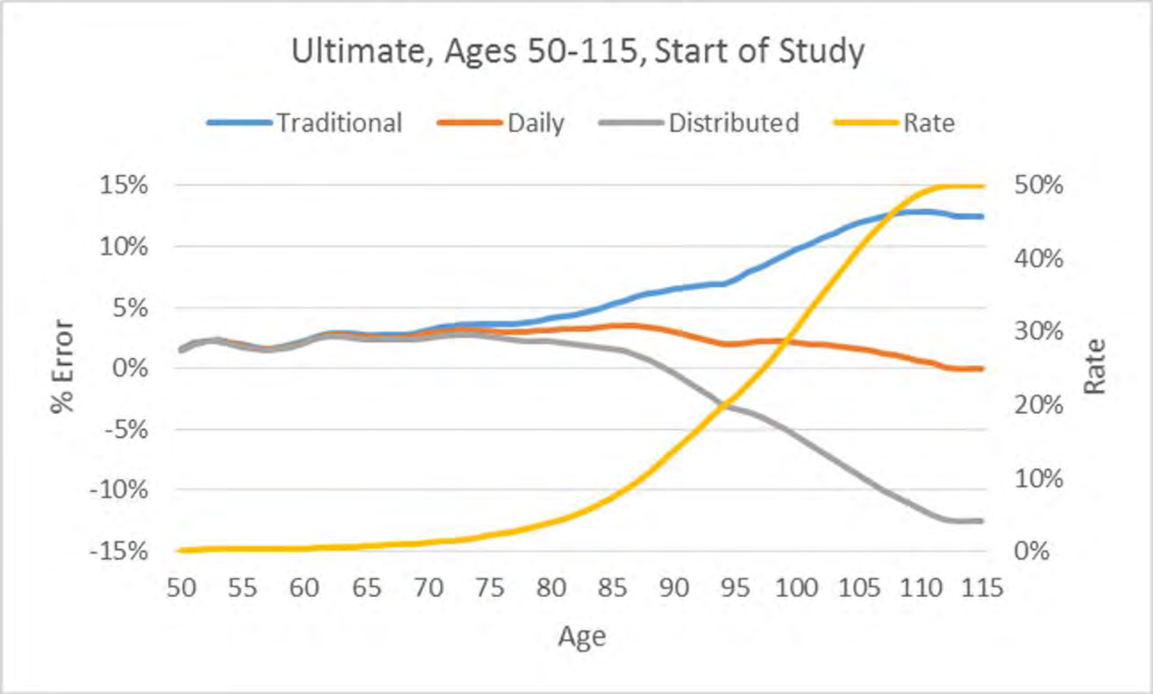
Half Year Rates



Half Year Rates



Rate Errors At Start Of Study



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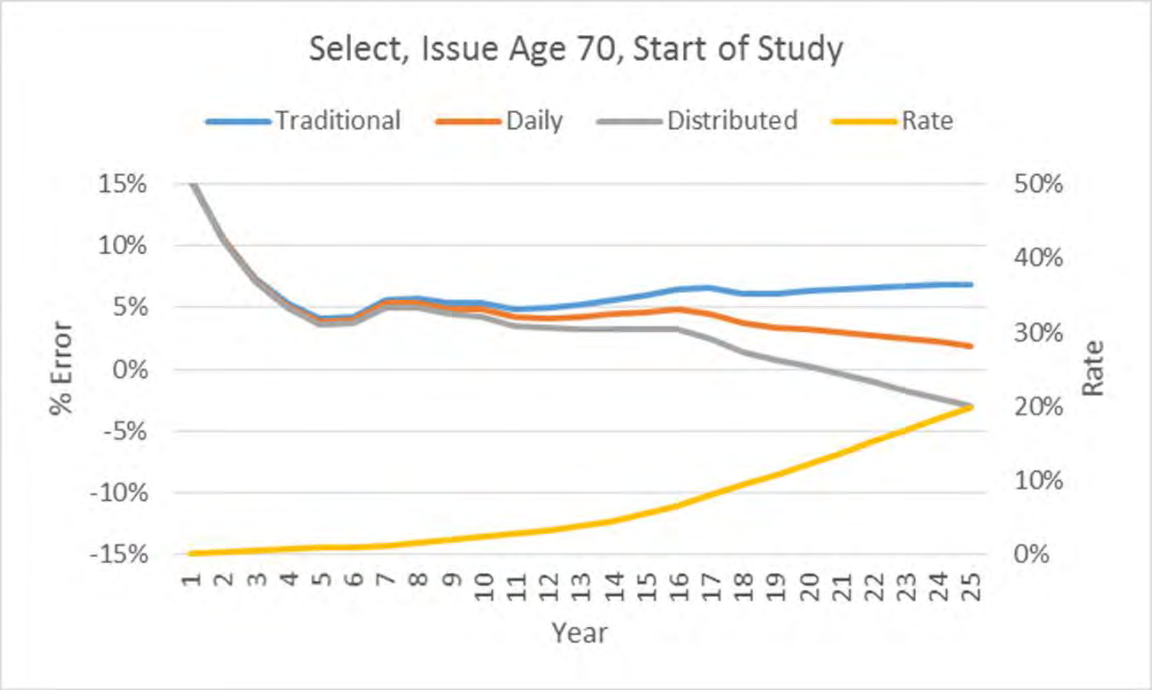

Study Errors

- The method errors will occur for a single cohort, i.e. the set of lives born in the same year, that will contribute to the same ages at the same time in the study.
- The study error for a single age in a study will accumulate the exposure-weighted method errors from multiple consecutive cohorts that contribute to the age.
- For a single age, if the cohorts are equal in size with the same anniversary distribution, the method error for the cohort at the start of the study will be equal in size and opposite in sign to the error for the cohort at the end of the study.

Study Errors – Single Cohort

- Study Period, 2012-14, Lives Born 1941.
- Errors by study year and in total.

Rate Errors by Study Year and Age						
Age	2011	2012	2013	2014	2015	Total
70	$\varepsilon_{70,½}$	$\varepsilon_{70½,½}$				$\varepsilon_{70½,½}$
71		$\varepsilon_{71,½}$	$\varepsilon_{71½,½}$			0
72			$\varepsilon_{72,½}$	$\varepsilon_{72½,½}$		0
73				$\varepsilon_{73,½}$	$\varepsilon_{73½,½}$	$\varepsilon_{73,½}$

- $\varepsilon_{x,½} < 0, \varepsilon_{x+½,½} > 0$ (increasing rates)
- $\varepsilon_{x+t,½} = -\varepsilon_{x,½}$

Study Errors – Four Cohorts

- Study Period, 2012-14, Lives Born 1941-44.
- Cohorts equal in size, homogeneous population.

Rate Errors by Cohort Year and Age					
Age	1941	1942	1943	1944	Total Rate Error
67				$\varepsilon_{67\frac{1}{2},\frac{1}{2},1}$	$\varepsilon_{67} = \varepsilon_{67\frac{1}{2},\frac{1}{2},1}$
68			$\varepsilon_{68\frac{1}{2},\frac{1}{2},1}$	0	$\varepsilon_{68} = \varepsilon_{68\frac{1}{2},\frac{1}{2},1}/3$
69		$\varepsilon_{69\frac{1}{2},\frac{1}{2},1}$	0	0	$\varepsilon_{69} = \varepsilon_{69\frac{1}{2},\frac{1}{2},1}/5$
70	$\varepsilon_{70\frac{1}{2},\frac{1}{2},1}$	0	0	$\varepsilon_{70,\frac{1}{2},4}$	$\varepsilon_{70} = 0$
71	0	0	$\varepsilon_{71,\frac{1}{2},3}$		$\varepsilon_{71} = \varepsilon_{71,\frac{1}{2},3}/5$
72	0	$\varepsilon_{72,\frac{1}{2},2}$			$\varepsilon_{72} = \varepsilon_{72,\frac{1}{2},2}/3$
73	$\varepsilon_{73,\frac{1}{2},1}$				$\varepsilon_{73} = \varepsilon_{73,\frac{1}{2},1}$

Study Errors – Full Exposure Ages

- In an N year study, each full-exposure age has $N + 1$ cohorts, $n = 1, N + 1$.
- Study Error
  - $\varepsilon_x = \left(E_{x+½,½,1}\varepsilon_{x+½,½,1} + E_{x,½,N+1}\varepsilon_{x,½,N+1}\right)/E_x$
- Equal Cohorts
  - $\varepsilon_x = 0$
- Estimate study error from
  - CalYr Rate – PolYr Rate
  - Error model and exposure distribution.
- Simple increasing cohort model developed to test the relationship between method and study errors.

Increasing Cohort Model (Trad Method)

1 Year Study	Range	Rate	% Method Error	% Study Error (Cohort Incr.)	
				10%	100%
Renewal Lapse	Year 10	50%	100%	-5%	-33%
Sel. Mortality Yr 1	Age 50,90	0.1%, 2%	10%, 32%	-0.5%, -1.5%	-3%,-11%
Ult. Mortality	Age 50-113	0.2%, 50%	2%, 13%	-0.1%, -0.4%	-0.5%, -3%

3 Year Study	Rate	Rate	% Method Error	% Study Error (Cohort Incr.)	
				30%	300%
Renewal Lapse	Year 10	50%	100%	-4%	-20%
Sel. Mortality Yr 1	Age 50,90	0.1%, 2%	10%, 32%	-0.5%, -1.5%	-2%,-6%
Ult. Mortality	Age 50-113	0.2%, 50%	2%, 13%	-0.1%, -0.4%	-0.3%, -1.7%

Conclusions

- Given exposure by study year and policy year, both calendar year and policy year study periods can be examined at the results stage.
- So a calendar year study period could be used if appropriate and conditions allow, otherwise a policy year study period.
- Depending on error tolerances and population distributions (i.e. cohort, age/year), calendar year study periods appear robust (errors < 3%) except for the most extreme distributions particularly with high rates.
- Rate distributions over the policy year will contain full method errors.

Questions?

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