SOCIETY OF ACTUARIES

Advanced Topics in General Insurance

Exam GIADV

Date: Thursday, May 6, 2021

INSTRUCTIONS TO CANDIDATES

General Instructions

1. This examination has 8 questions numbered 1 through 8 with a total of 40 points.

   The points for each question are indicated at the beginning of the question.

2. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions provided in this document.

Written-Answer Instructions

1. Each question part or subpart should be answered either in the Word document or the Excel file as directed. Graders will only look at work in the indicated file.

   a) In the Word document, answers should be entered in the box marked ANSWER. The box will expand as lines of text are added. There is no need to use special characters or subscripts (though they may be used). For example, β_1 can be typed as beta_1 and σ^2 can be typed as sigma^2.

   b) In the Excel document formulas should be entered. Performing calculations on scratch paper or with a calculator and then entering the answer in the cell will not earn full credit. Formatting of cells or rounding is not required for credit. Rows can be inserted to the answer input area as required to provide space for your answer.

   c) Individual exams may provide additional directions that apply throughout the exam or to individual items.

2. The answer should be confined to the question as set.

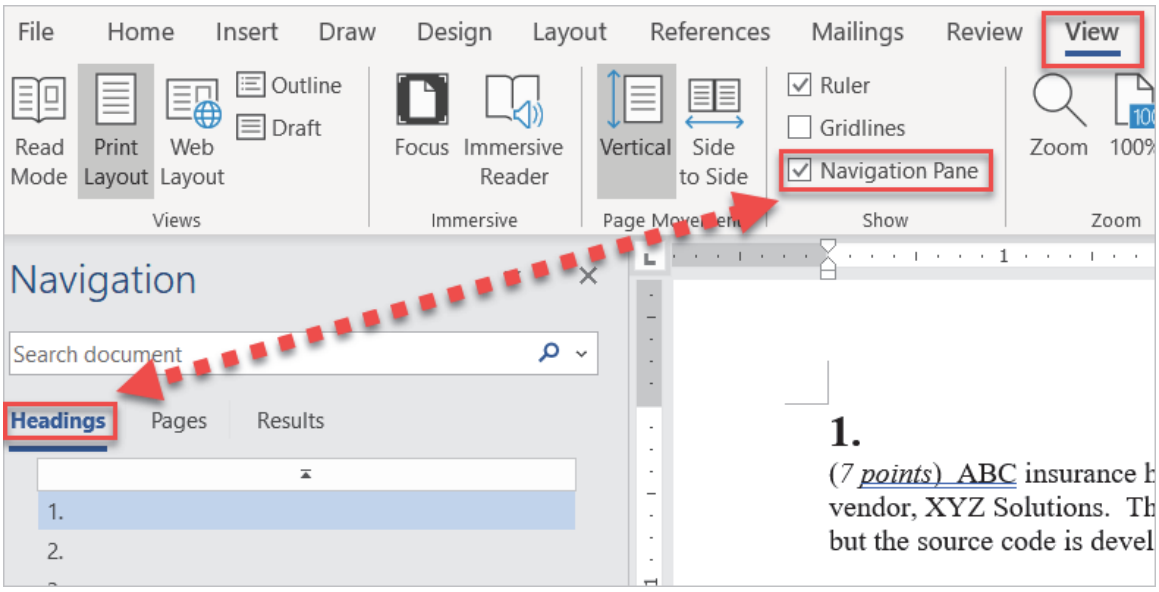
3. Prior to uploading your Word and Excel files, each file should be saved and renamed with your five-digit candidate number in the filename.

4. The Word and Excel files that contain your answers must be uploaded before time expires.

Navigation Instructions

Open the Navigation Pane to jump to questions.

Press Ctrl+F, or click View > Navigation Pane:



1.

(*5 points*) Casualty R Us Reinsurance Company is evaluating a proposed casualty per occurrence excess treaty.

You are developing increased limits factors (ILFs) using a lognormal model.

The underlying losses have the following characteristics:

- Mean 300
- Standard Deviation 1,200

The estimated parameters of the lognormal distribution based on the method of moments are:

- mu (μ) 4.287
- sigma (σ) 1.683

(a) (*1 point*) Demonstrate that this is true.

Provide the response for this part in the Excel spreadsheet.

The limited expected loss function for a lognormal distribution is given by:

$$E[y;L] = exp\left(\mu + \frac{\sigma^2}{2}\right) \cdot \Phi\left(\frac{ln(L) - \mu - \sigma^2}{\sigma}\right) + L \cdot \left[1 - \Phi\left(\frac{ln(L) - \mu}{\sigma}\right)\right]$$

$\Phi(x)$ is the cumulative distribution function for a standard normal variable, N(0,1). It can be produced by the Excel function NORM.S.DIST(*x*,TRUE).

The following increased limits factors were generated:

Policy Limit	Increased Limits Factor
500	1.00
1,000	1.28
1,500	1.44
2,000	1.53

(b) (*1.5 points*) Demonstrate that the ILF at policy limit 1,500 is 1.44.

Provide the response for this part in the Excel spreadsheet.

1. Continued

You are pricing the layer 1,000 excess of 500.

The following information has been provided:

Subject Premium	Underlying Limit	Policy Limit
2,000	0	1,000
2,500	0	1,500
4,000	500	1,000
4,500	500	1,500

The expected loss ratio is 55%.

(c) (*2.5 points*) Calculate the expected losses in the layer using an exposure rating approach.

Provide the response for this part in the Excel spreadsheet.

2.

(*4 points*) Discounted cash flow analyses may use a net present value (NPV) approach or the internal rate of return (IRR) method. However, there is a problem that exists with the IRR method.

(a) (*1 point*) Describe the problem with the IRR method.

ANSWER:

You are calculating the underwriting profit margin (UPM) for a one-year policy using the Risk Adjusted Discount Technique with the following assumptions:

- The premium will be collected at policy inception.
- Expenses of 24 will be paid as follows:
  - 35% paid six months before policy inception; and
  - 65% paid at policy inception.
- Losses are expected to be 120 and will be paid as follows:
  - 40% paid nine months after policy inception; and
  - 60% paid at policy expiration.
- The tax rate on all income is 25% and taxes will be paid at policy expiration.
- Equity of 90 supports the policy from policy inception to policy expiration.
- The risk-free rate is 3.4%.
- The risk-adjusted rate for losses is 0.6%.

(b) (*2.5 points*) Calculate the premium for this policy.

Provide the response for this part in the Excel spreadsheet.

(c) (*0.5 points*) Calculate the UPM for this policy.

Provide the response for this part in the Excel spreadsheet.

3.

(*4 points*) You are given the following data extracted from a triangle of cumulative paid losses:

Accident Year	From (months)	To (months)	Increment	Diagonal Age	Accident Year Total
2017	0	12	2,500	48	5,000
2017	12	24	1,800	48	5,000
2017	24	36	500	48	5,000
2017	36	48	200	48	5,000
2018	0	12	4,100	36	7,000
2018	12	24	2,000	36	7,000
2018	24	36	900	36	7,000
2019	0	12	4,600	24	6,800
2019	12	24	2,200	24	6,800
2020	0	12	5,300	12	5,300

You are also given the following onlevel premiums:

Accident Year	Onlevel Premium
2017	10,000
2018	12,000
2019	15,000
2020	18,000

You apply Clark's stochastic reserving model using the Cape Cod method and an exponential distribution with cumulative distribution function $G(x) = 1 - e^{-x/\theta}$ where x is in months.

The maximum likelihood estimate of *ELR* is 0.5424 and of θ is 8.104.

(a) (*1 point*) Estimate the scale factor, σ^2.

Provide the response for this part in the Excel spreadsheet.

(b) (*1.5 points*) Estimate the process standard deviation of the loss reserve for all accident years combined.

Provide the response for this part in the Excel spreadsheet.

3. Continued

In 2022, the premium is expected to be 20,000.

(c) (*0.5 points*) Estimate the expected loss for 2022.

Provide the response for this part in the Excel spreadsheet.

(d) (*0.5 points*) Estimate the coefficient of variation due to process variance for the 2022 loss.

Provide the response for this part in the Excel spreadsheet.

The covariance matrix of the estimates of *ELR* and θ, respectively, is:

$$\begin{pmatrix} 0.00147 & 0.01112 \\ 0.01112 & 0.64988 \end{pmatrix}$$

(e) (*0.5 points*) Estimate the coefficient of variation due to parameter variance for the 2022 loss.

Provide the response for this part in the Excel spreadsheet.

4.

(*9 points*) You are interested in determining a model for loss development. The triangle of incremental loss data you are working with, by accident year (AY) and development year, is:

	Development Year						
AY	1	2	3	4	5	6	7
1	5,012	3,257	2,638	898	1,734	2,642	1,828
2	106	4,179	1,111	5,270	3,116	1,817	
3	3,410	5,582	4,881	2,268	2,594		
4	5,655	5,900	4,211	5,500			
5	1,092	8,473	6,271				
6	1,513	4,932					
7	557						

You begin by regressing the incremental losses against the previous cumulative losses. The results are presented below, where the constant is denoted by *a* and the factor by *b*.

	1 to 2	2 to 3	3 to 4	4 to 5	5 to 6
a	5,892	-970	4,167	2,662	48,608
b	-0.181	0.562	-0.059	-0.014	-3.395

The triangle of fitted incremental loss data is:

	Development Year						
AY	1	2	3	4	5	6	7
1	-	4,987	3,674	3,518	2,496	2,642	-
2	-	5,873	1,437	3,846	2,512	1,817	
3	-	5,277	4,080	3,342	2,435		
4	-	4,871	5,519	3,229			
5	-	5,695	4,402				
6	-	5,619					
7	-						

(a) (*1 point*) Compute the sum of squared errors (SSE).

Provide the response for this part in the Excel spreadsheet.

4. Continued

You now wish to use regression to fit the following alternative models:

(i) Constant only

(ii) Factor only

(iii) Factor only, with each observation weighted by the reciprocal of the previous cumulative losses

(b) (*6 points*) Construct the fitted triangle and compute the SSE, for each of the three alternative models.

Provide the response for this part in the Excel spreadsheet.

(c) (*1 point*) Compute one test statistic, based on the SSE, for each model.

Provide the response for this part in the Excel spreadsheet.

(d) (*0.5 points*) Identify the best model based on the value of this test statistic for each model.

Provide the response for this part in the Excel spreadsheet.

(e) (*0.5 points*) Describe the correlation structure of the best model.

Provide the response for this part in the Excel spreadsheet.

5.

(*5 points*) A risk load can be calculated for an account during build-up and on renewal. Consider a portfolio with two accounts, an older account X and a newer account Y.

Under the Marginal Variance method for calculating risk loads, the risk load for account X on build-up is different from the risk load for account X on renewal.

(a) (*0.5 points*) Identify which risk load is larger.

ANSWER:

(b) (*1 point*) Explain why there is this difference.

ANSWER:

An insurer is renewing these two accounts, X and Y, each of which is exposed to five independent loss events as follows:

Event (*i*)		**Loss for Account**	
i	***p*(*i*)**	**X**	**Y**
1	1.5%	15,000	500
2	1.0%	9,000	2,000
3	2.5%	6,500	2,500
4	3.0%	4,500	5,000
5	2.0%	1,000	10,000

- $p(i)$ represents the probability of Event i.
- The risk load multiplier, λ, is 0.000045.

(c) (*3.5 points*) Calculate the renewal risk load for each account using the following methods:

(i) Marginal Variance

(ii) Shapley

Provide the response for this part in the Excel spreadsheet.

6.

(*4 points*) You are calculating a risk margin for claim liabilities using the methodology set out in "A Framework for Assessing Risk Margins."

The risk margin is to be calculated at the 80% adequacy level and is to be based on the following sources of uncertainty, which are assumed to be mutually independent:

		Coefficients of Variation		
Line of Business	**Claim Liabilities**	**Independent Risk**	**Internal Systemic Risk**	**External Systemic Risk**
Motor	8,000	6.0%	5.0%	4.5%
Property	4,000	10.0%	9.0%	4.0%
Total	**12,000**	**5.2%**	**5.0%**	**3.3%**

- The correlation between lines for internal systemic risk was assessed at 25%.
- The correlation between lines for external systemic risk was assessed at 0%.
- Claims are assumed to be normally distributed.
- The *z*-value of the 80th percentile of the normal distribution is 0.8416.

(a) (*1 point*) Verify that the internal systemic risk coefficient of variation is 5.0% (rounded to one decimal place).

Provide the response for this part in the Excel spreadsheet.

(b) (*1 point*) Calculate the aggregate coefficient of variation for both lines combined.

Provide the response for this part in the Excel spreadsheet.

(c) (*1 point*) Calculate the amount of the risk margin at the 80% adequacy level.

Provide the response for this part in the Excel spreadsheet.

An alternative to the normal distribution assumption is the lognormal distribution.

(d) (*1 point*) Provide one argument in favor of and one argument against assuming the lognormal distribution for claims in this situation.

ANSWER:

7.

(*5 points*) You are working with an empirical distribution based on the following sample of ten losses:

20, 30, 60, 90, 110, 110, 130, 160, 240, 400

(a) (*1 point*) Calculate the expected payment per loss for a policy with a limit of 100.

Provide the response for this part in the Excel spreadsheet.

The equivalence between the size mode of summation and the layer mode of summation for the layer from 100 to 200 can be expressed as

$$\int_{100}^{200} x dF(x) + 200G(200) - 100G(100)$$

$$= \int_{100}^{110} G(x)dx + \int_{110}^{130} G(x)dx + \int_{130}^{160} G(x)dx + \int_{160}^{200} G(x)dx$$

where $F(x)$ is the cumulative distribution function and $G(x) = 1 - F(x)$.

(b) (*2 points*) Calculate the value of each term in this equation.

Provide the response for this part in the Excel spreadsheet.

(c) (*1 point*) Calculate increased limits factors for 110, 130, 160 and 200 with a basic limit of 100.

Provide the response for this part in the Excel spreadsheet.

(d) (*1 point*) Demonstrate that these increased limits factors are consistent.

Provide the response for this part in the Excel spreadsheet.

8.

(*4 points*) ABC Reinsurance Company is pricing the 2021 renewal of its proportional reinsurance treaty with Ceding Insurance Company.

The treaty losses have the following loss distribution:

Loss	Probability
0	9.16%
40,000	14.65%
110,000	19.54%
180,000	19.54%
250,000	15.63%
320,000	10.42%
390,000	5.95%
400,000	5.11%

The 2021 treaty premium is 331,000.

The treaty has a sliding scale commission:

Loss Ratio	Commission
30% or below	45%
30%-50%	Sliding 0.5:1
50%-60%	Sliding 1:1
60% or above	25%

ABC expenses are 8.0% of ceded premium.

(a) (*0.5 points*) Show that with the expected loss ratio of 54.0%, the 2021 treaty profit is 7.0% of ceded premium.

Provide the response for this part in the Excel spreadsheet.

(b) (*1.5 points*) Show that using the loss distribution above, the expected 2021 treaty profit is 3.6% of ceded premium.

Provide the response for this part in the Excel spreadsheet.

(c) (*0.5 points*) State whether or not the sliding scale commission structure is "balanced." Justify your answer.

Provide the response for this part in the Excel spreadsheet.

8. Continued

The treaty has a carryforward provision.

The loss ratio on the 2020 treaty was 75.5%.

(d) (*1 point*) Recalculate the expected 2021 treaty profit from (b) as a percentage of ceded premium, allowing for the loss ratio in 2020.

Provide the response for this part in the Excel spreadsheet.

Another approach to assessing the effect of a carryforward provision is to look at the "long run" of the contract.

(e) (*0.5 points*) State two problems with this approach.

ANSWER:

****END OF EXAMINATION****