ILA LAM Model Solutions
Spring 2023

1.   **Learning Objectives:**

2.   The candidate will understand and be able to assess issues and concerns common to actuarial models and their development and management.

4.   The candidate will understand the basic design and function of Economic Scenario Generators and Equity Linked Insurance Models.

Learning Outcomes:

(2h)   Describe and evaluate the guidance in the Actuarial Standards of Practice.

(2k)   Describe and evaluate considerations related to modeling investments, discount rates, inflation and catastrophic mortality.

(4a)   With respect to Economic Scenario Generators:
- Describe the need for ESGs and explain the structure of ESG models and components.
- Describe and apply basic default free interest rate models, including one-factor continuous time models.
- Assess the propriety of a particular ESG model and related assumptions for particular applications.

Sources:

The Effect of Deflation or High Inflation on the Insurance Industry, 2012 (excluding pp. 11-14)

Interesting Challenges for Insurers, Product Matters, Jun 2012

Economic Scenario Generators: A Practical Guide, 2016, Ch . 1, 2, 4.1, 5, 6, 9, 10, 11.1 & 11.3

ASOP 56: Modeling, Sections 3 & 4

Commentary on Question:
Commentary listed underneath question component.

1. Continued

Solution:

(a) For each scenario listed below:

(i) Assess the impact on your company

(ii) Recommend a risk mitigation strategy if appropriate

(iii) Discuss any potential disadvantages of the risk mitigation strategy proposed in (ii)

	Economic Scenario	Asset / Liability
A	High Inflation	Whole Life Insurance with a fixed policy loan rate
B	Deflation	Universal Life with minimum crediting rate guarantee
C	Rising Interest Rates	Universal Life with minimum crediting rate guarantee
D	High Inflation	Fixed annuities with cost-of-living adjustments indexed to inflation
E	High Inflation	Long-term fixed bonds

Commentary on Question:
The question was testing the ability of candidates to identify the risk of various inflationary and interest rate scenarios on various Products. Candidates needed to identify valid methods of mitigation for the product or the company. Most candidates were able to identify the risk facing the product or company in scenario A, B and C. Better candidates were able to identify the risks in scenario D and E. Credit was given where reasonable mitigation strategies were proposed. A number of candidates gave generic risks of the mitigation strategies, better candidates were able to give specific risks of the mitigation strategies proposed

Scenario A
Risk: Policyholders are likely to take out loans to invest elsewhere if the fixed loan rate is lower than prevailing interest rates. This will force your company to sell assets to meet outgoing cashflows. In other words, disintermediation risk
Mitigation: Don't offer fixed policy loan rates. Instead, index rates (e.g. central bank rate + x%).
However, it may not be possible to index rates that were contractually fixed.

Scenario B
Risk: Investment returns may fall short of the guarantees, creating significant risk for the company
Mitigation Reduce the minimum guaranteed rates on fixed products, particularly UL; can retain flexibility via the current credited rate mechanism to offer higher rates if interest rates rise. Develop triggers that are linked to the risk of deflation
Disadvantage: consumer dissatisfaction if eliminate guarantees

1. Continued

Scenario C
Risk: Disintermediation. Policyholders likely to lapse or withdraw as much cash value as possible to seek higher returns elsewhere. Portfolio rate will be dragged down by Inforce assets purchased at lower rates and will not keep up with new money yields.
Mitigation: Increase credited rates as much as possible.
Disadvantage is compressed spreads.

Scenario D
Risk: Fully indexed for inflation could be cost prohibitive if the risk of hyperinflation is reflected
Mitigation: Add a cap for unexpected exposure during sustained high inflation/Invest into index linked securities to hedge against the impact of inflation
Disadvantage is there is counterparty risk introduced/ It may not be possible to fully hedge the risk of sustained or hyper inflationary effects

Scenario E
Risk: Bond value decreases significantly. They create drag on income and become difficult to sell.
Mitigation: Reinvest in short-term assets to reduce impact on inflation.
Disadvantage is reduced investment income

(b) You have been asked to review your company's Economic Scenario Generator (ESG) regarding its suitability to simulate future interest rate paths including inflation.

Critique the following statements:

A. *The applications of the ESG are primarily focused on the interaction of interest rate changes and policyholder behavior so its use is limited to liability valuation. It is not suitable for stress testing.*

B. *To model the relationship between inflation and interest rates your company uses parameters based on historical inflation data from the last 30 years.*

C. *For calibrating the parameters within an ESG, making use of a cascade structure where interest rates are at the top of the cascade with inflation below would imply that interest rate changes cause inflation.*

1. Continued

Commentary on Question:
Candidates were asked to comment on various aspects of an ESG and respond to statements about the structure and purpose of an ESG. Candidates who got full credit correctly challenged the validity of statements and explained why the statements were inaccurate or only partially true. Most candidates could identify that Statement A was not completely true, but few were able to acknowledge the uses of the ESG for life insurance liabilities correctly. For scenario B, some candidates confused the question as being about real-world vs risk neutral ESGs versus discussing the broader issues arising from the time period selected to calibrate parameters. Scenario C was poorly answered – most candidates were unable to recognize that the cascade structure for calibration was not meant to imply a direct causal relationship of variables.

Statement A
While it's true that applications of ESGs for life insurance liabilities are primarily focused on the interaction of interest rate changes and policyholder behavior regarding lapses and other optionality, this does NOT mean that ESGs are only suitable for valuation. Other uses include effective duration analysis, stress testing, economic capital (EC) and strategic asset allocation (SAA).

Statement B
This is incorrect due to Insufficient data - A model of future inflation rates should not be parameterized based solely on the levels of inflation experienced in the last few decades. A longer time horizon including the deflation periods and high inflation periods of the past should be used, as well as consideration of developments in other countries that have faced similar economic conditions, needs to be reflected in any inflation model.

Statement C
Not true. The sequence of the cascade is not intended to reflect any cause-and-effect relationship but rather is designed to ease the calibration process while allowing variables to be appropriately correlated.
In the cascade structure a variable lower in the cascade is only affected by variables above it and its prior values. For example, making use of a cascade structure where interest rates are at the top of the cascade with inflation below would not imply that interest rate changes cause inflation. It would, however, allow the simulation of interest rates and inflation to have an appropriate relationship, so that when interest rates are high, inflation will tend to be high, and vice versa.

(c)   Identify four relevant recommended practices from *ASOP 56, Modeling,* that should be considered when relying on external experts to develop your company's ESG.

1. Continued

Commentary on Question:
Candidates were asked to recall the recommended practices when relying on external experts as per ASOP 56. Candidates scored poorly on this question with few being able to comprehensively recall the material. Better candidates could identify the need to document reliance and to establish expertise and relevance

In determining the appropriate level of reliance, the actuary may consider the following:
a. whether the individual or individuals upon whom the actuary is relying on are experts in the applicable field;
b. The extent to which the model has been reviewed or validated by experts in the applicable field, including known material differences of opinion among experts concerning aspects of the model that could be material to the actuary's use of the model.
c. whether there are industry or regulatory standards that apply to the model or to the testing or validation of the model, and whether the model has been certified as having met such standards; and
d. whether the science underlying the expertise is likely to produce useful models for the intended purpose.

When relying on experts, the actuary should disclose the extent of such reliance.

2.   **Learning Objectives:**

5.   The candidate will understand the role of the Investment Actuary and the Portfolio Management Process in the Life Insurance company context, as well as the common forms of Fixed income securities and their uses, and the methods and processes used for evaluating portfolio performance and asset allocation.

Learning Outcomes:

(5f)   Describe and apply methods and processes for evaluating portfolio performance, including performance attribution, sources of earnings analysis on investment income, benchmarks, metrics, and risk adjusted performance appraisals (including total return vs reported earnings).

Sources:

Managing Investment Portfolios, Maginn, John L. and Tuttle, Donald L., 3rd Edition, 2007 - Ch. 5: Asset Allocation (sections 2-4)

Managing Investment Portfolios, Maginn, John L. and Tuttle, Donald L., 3rd Edition, 2007 - Ch. 6: Fixed-Income Portfolio Management (sections 1-5)

Commentary on Question:

The purpose of this question is to test candidates' ability to evaluate portfolio performance and asset allocation. To receive full credit, candidates would need to recognize the pros and cons of different alternatives and make reasonable recommendations with regard to the case presented.

Candidates generally did well in the calculation part of this question. For essay answers, most candidates were able to identify reasonable areas of consideration. Stronger candidates were able to comment on different aspects of each asset allocation method and justify why or why not it is appropriate to the company's information presented in the case.

Solution:

(a)

(i)   Calculate the cash required for rebalancing to maintain the initial portfolio dollar duration. Show all work.

(ii)   Discuss considerations when rebalancing a portfolio when using an immunization strategy.

2. Continued

Commentary on Question:
Most candidates did well on part i and were able to calculate the final cash required. Some candidates were confused between dollar-duration vs weighted-average duration of the portfolio, which only partial marks were awarded.
On part ii, many candidates had challenges to identify considerations on portfolio rebalancing and tend to focus on commenting the pros and cons of classic immunization strategy, which is not the primary focus of this question. Stronger candidates were able to comment on why rebalancing is required and what are some key areas of considerations in practice, with some supporting commentary on immunization strategy.

(i)
Refer to Excel file

(ii)
Rebalancing is an important part of an immunization strategy. The market yield rates will fluctuate over the investment horizon as a result the duration of the portfolio will change
When rebalancing, a company needs to consider both the cost and benefits. More frequent rebalancing increases the transaction costs, but less frequent rebalancing causes the duration to wander from the target duration. The company will need to strike the optimal balance based on risk appetite and business objective.
As classical immunization techniques are usually subject to limitations, which immunization technique to adopt should also be taken into consideration when rebalancing.
Liquidity of securities used to construct an immunized portfolio should also be considered as illiquid securities involve high transaction costs.

(b)   Critique each of the proposed changes to FGE's investment strategy:

   A   FGE will use a dynamic asset liability approach

   B   FGE will change its current target allocation of 75% fixed income/15% equities/10% cash to a new target allocation of 20% fixed income/75% equities/5% cash

   C   FGE will use a strategic asset allocation to exploit short term opportunities in the market

   D   FGE will offer bonuses to portfolio managers who are able to identify strategies that improve returns

2.   Continued

Commentary on Question:
Most candidates had some challenges earning full credits in this question.
In part A, some candidates seemed to mix up the dynamic approach vs dynamic hedging, and many candidates were not able to comment on asset liability approach vs asset-only approach.
In part B, many candidates focuses their comments on the % change in equities without much comment on the % change in the other two asset classes.
In part C and D, most candidates were able to identify "what is wrong" with the statements, and candidates who were able to make recommendations with justifications earned full credits.

A.
A dynamic strategy is more costly, but is worthwhile when there are significant future liabilities, such is the case here; ALM is a good fit given the low risk tolerance of FGE. Asset-liability approach is more appropriate than an asset-only approach given FGE carries life insurance products and has long term liabilities, and the penalty for not meeting its cash flow obligation is high. It would be better to explicitly model liability cash flows and align asset strategy accordingly.

B.
The large proportion of fixed income in the initial target is high but offers stable cash inflows. Given the long duration of the company's liabilities this can be justified.
The shift from 15% to 75% equities may generate higher return but is not in line with the stated very low risk tolerance. The company should set the target allocation with the company risk appetite in mind. They should also use more specific asset classes for their targets.
A 10% cash balance is high but allows extra liquidity when it comes to the company's liability management. The change to 5% will reduce liquidity and should be justified against the company's risk appetite, particularly on liquidity risk.

C.
The statement is incorrect. Strategic asset allocation is not the exploitation of short-term opportunities. Strategic asset allocation is when return objectives, risk tolerance and investment constraints are integrated with long-term capital market expectations.
Instead, tactical asset allocation is more often used to exploit short-term opportunities involving making short-term adjustments to asset-class weights based on short-term predictions of relative performance among asset classes. Strategic asset allocation would be an appropriate fit for the company given the long-term nature of its insurance policy liabilities and its conservative risk appetite.

2. Continued

D.
Offering bonuses to portfolio managers purely based on returns would not be an appropriate strategy for the company.
This approach will encourage portfolio managers to take riskier strategies in order to improve returns, which goes against the company's risk appetite.
Implementing strategic asset allocation will be a better option in terms of controlling the systemic risk.

(c) Recommend which asset allocation FGE should choose, using Roy's safety-first criterion. Show all work and justify your answer.

Commentary on Question:
Candidates in general did well on this question. Most candidates were able to correctly calculate Roy's safety-first criterion and risk-adjusted return, and make appropriate recommendations based on the results. Some candidates made their recommendations based solely on risk-adjusted returns, of which only partial marks were given.

Refer to Excel file for model solution

3.

Learning Objectives:

3. The candidate will understand the principles of Asset-liability Management ("ALM"), and be able to describe and evaluate various techniques for addressing the mitigation of risk.

Learning Outcomes:

(3a) With respect to Asset-Liability Models:
- Describe and apply the fundamental elements of the theory and practice of ALM in an insurance company, including assessing the dangers of mismatched assets and liabilities.
- Describe and demonstrate how ALM can be used to identify and manage product and asset risks, including:
  o Major product risks for which ALM can be a useful tool for their management.
  o Using ALM as a means to manage interest rate risk, equity risk, and risks from optionality.
- Describe how common insurance contracts and variations generate embedded options in an insurer's balance sheet, and assess basic strategies for managing exposures created by such embedded options.
- Describe and apply the basic concepts of cash flow matching, immunization, duration/convexity matching, segmentation.
- Describe and apply Key Rate Durations (KRD) and their use in evaluating interest rate sensitivities of portfolios, including understanding the derivation of KDRs, the profiles of KDRs for selected major asset types, and assessing KRDs in a portfolio context.
- Describe and evaluate the Goldman Sachs' ALM/Strategic Asset Allocation approach for integrating ALM into an enterprise's risk and financial management framework.
- Describe and evaluate ALM modeling considerations in the context of modeling risk aggregation, dependency, correlation of risk drivers and diversification.

Sources:

LAM-118-14: Revisiting the Role of Insurance Company ALM w/in a RM Framework

LAM-117-14: Key Rate Durations: Measures of Interest Rate Risk

Commentary on Question:
This goal of this question is to test the candidate's understanding of the steps required to establish a process for strategic asset allocation. It also tests the candidate's ability to interpret various efficient frontiers, and the calculation and use of key rate durations.

3. **Continued**

Solution:

(a)   Critique the following statements:

A.   Given our company's historical success when limiting duration mismatch to within 0.5 years and limiting portfolio exposure to alternative assets to 10%, we will hold these constraints constant as we explore SAA.

B.   Adding new asset classes will allow us to better diversify risks and optimize efficiency by considering possible correlations between various asset classes and correlations with our liabilities.

C.   When building a replicating portfolio, we should prioritize matching the key rate duration (KRD) profile of liabilities instead of focusing only on minimum interest rate guarantees.

D.   Given surplus volatility was the most severe impact of the recent recession, the SAA process will focus only on minimizing surplus volatility.

E.   With an objective of closely matching the cash flows or interest rate duration of our liabilities, we maintain a separate investment portfolio to back the reserves for each of our major liability types.

F.   A model should be built which seeks to maximize return for a given level of surplus volatility while factoring in our chosen constraints. This will provide an efficient frontier that can be used to determine our risk appetite.

Commentary on Question:
Partial points were granted if the candidates opined on the accuracy of each statement. Full marks were only given when candidate critiqued each statement and defended their conclusion with information that aligns with source material.

Most of the candidates were able to receive partial credits

A. This statement is incorrect. While it is appropriate to establish constraints to maintain for SAA such as the items listed, it is expected that building an SAA process will require an iterative approach with targets and constraints. The company should allow flexibility around their constraints, regardless of historical precedent.

3. Continued

> *B.* This statement is correct. Expanding asset classes considered for portfolio will increase ability to optimize both diversification and efficiency around targets. Understanding the correlations is key to accomplishing this, as the company notes.
>
> *C.* This statement is incorrect. Including KRDs when building a replicating portfolio is appropriate, however both consideration of duration matching and risks associated with interest rate guarantees should be included in focus for risk minimization.
>
> *D.* This statement is incorrect. While having a key metric in focus such as surplus volatility is appropriate for assessing performance, including multiple risk metrics allows for a complete understanding of the strategy's performance. The efficient frontier may vary under different metrics, and multiple perspectives should be accommodated.
>
> *E.* This statement is incorrect. Holistic ALR and SAA consider the entire asset portfolio in aggregate to first optimize risk-adjusted returns within capital constrains and risk tolerance levels while simultaneously determining the most effective constraint for ALM
>
> *F.* This statement is correct. To create an efficient frontier the model should be able to maintain established constraints in determining a portfolio that optimize risk and return

(b)

    (i)   Calculate the change in surplus under each shock. Show all work.

    (ii)   Assess if the investment strategy immunizes the company's surplus.

    (iii)   Identify key considerations if implementing a liquidity risk policy for this product.

Commentary on Question:
Many of candidates were able to calculate the surplus change under each shock correctly. Almost all the candidates concluded the investment strategy does not immunize the company's surplus. However only a few candidates commented on the shocks and investment strategies.
Most candidate received partial credits on part iii.

    (i)   % change in Liability $= [\,P^* - P\,]\,/\,P = \Delta P\,/\,P = \sum (-1) * D(i) * d(i)$
           New liability value_1 $= (1 + \%$ change in liability_1$) * P = [\,(1 + (-1.93\%))\,] * 100$ million $= \$98.08$

New liability value_2 = (1 + % change in liability_2) * P = [(1 + (-1.67%)] * 100 million = $97.39

New liability value_3 = (1 + % change in liability_3) * P = [(1 + (0.08%)] * 100 million = $100.084

% change in Asset = (-1) * [wgt1 * D1*d1 + wgt2 * D1*d1] = (-1) * [1/2 * 5yr * d1 + 1/2 * 10yr * d1]

New Asset value = (1 + % change in Asset) * P = (1 + Δ P / P) * 100 million

New Asset value_1 = [(1 + (-1.88%)] * 100 million = $98.125

New Asset value_2 = [(1 + (-3.25%)] * 100 million = $96.75

New Asset value_3 = [(1 + (-2.50%)] * 100 million = $97.5

	Scenario 1	Scenario 2	Scenario 3
Δ% in Liab	-1.925%	-2.615%	0.084%
New Liab	$ 98.08	$ 97.39	$ 100.0840
Δ% in Asset	-1.8750%	-3.2500%	-2.5000%
New Asset	$ 98.125	$ 96.75	$ 97.5000
Δ in Surplus	$ 0.050	$ (0.635)	$ (2.584)

(ii) Shock #1 is small parallel shifts to the yield curve; #2 is curvature shift of {1,0,1} where short rate drops and long rate raises. #3 is a steepness shift of {-1, 0,1} where short rate moves more than long rate;
As can been seen from part i's calculated results, surplus changes are much higher in non-parallel shifts (shock #2 and #3), than parallel shifts (shock #1). This indicates the assets and liabilities are not fully immunized against each other, causing the CFO's objective not being satisfied.
Since the yield curve rarely moves in parallel fashion, to minimize surplus movements caused by non-parallel shifts, portfolios that are immunized based on a KRD strategy is recommended since it doesn't need significant rebalancing of assets/liabilities. For example, trying to immunize the portfolio by investing the assets backing liabilities in a range of maturity levels (from 1yr to 10 yrs) could lower surplus movements.

(iii) Liquidity risk is both an asset and liability concern. The company needs to consider characteristics of both the annuity payments as well as the liability when setting a liquidity policy.
A company's strategic asset allocation and contingent liquidity planning should directly reflect the expected a contingent liquidity needs of its liabilities and potential sudden extreme shifts of liquidity in the financial market.
Because there is a cash flow timing mismatch of asset (5 and 10 year bonds) and liabilities (every year) this may lead to liquidity issues when selling or reinvesting bonds depending on shifts in the yield curve.
The company should have a written liquidity policy, a written liquidity stress management plan and should continually monitor the liquidity risk.

4.   **Learning Objectives:**
4.   The candidate will understand the basic design and function of Economic
        Scenario Generators and Equity Linked Insurance Models.

Learning Outcomes:
(4a)   With respect to Economic Scenario Generators:
- Describe the need for ESGs and explain the structure of ESG models and components.
- Describe and apply basic default free interest rate models, including one-factor continuous time models.
- Assess the propriety of a particular ESG model and related assumptions for particular applications.

(4b)   With respect to Equity-Linked models:
- Describe and apply methods for modeling long-term stock returns and certain guarantee liabilities (GMMB, GMDB, GMAB).
- Describe and evaluate the Actuarial and Hedging risk metrics for GMAB and GMDB models.
- Describe and apply methods for modeling Guaranteed annuity options and Guaranteed Minimum Income Benefits (GMIB), and EIA guarantees.

Sources:
Investment Guarantees, Hardy, Mary, 2003, Ch. 6: Modeling the Guarantee Liability

Investment Guarantees, Hardy, Mary, 2003, Ch. 7: A Review of Option Pricing Theory
(pp. 115-125)

Commentary on Question:
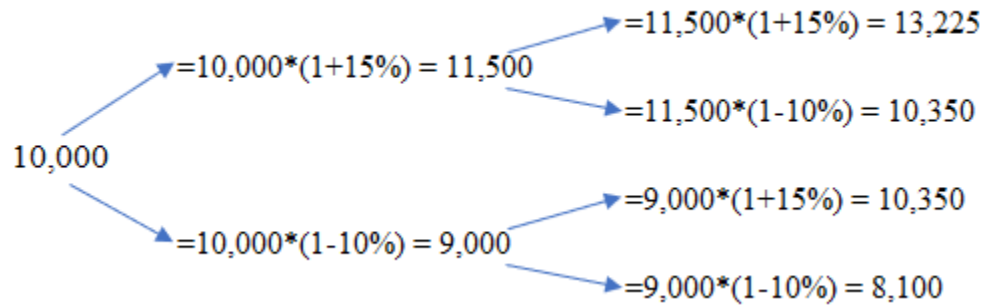*Commentary listed underneath question component.*

Solution:
(a)   Calculate all GMAB claims at the end of year 2.
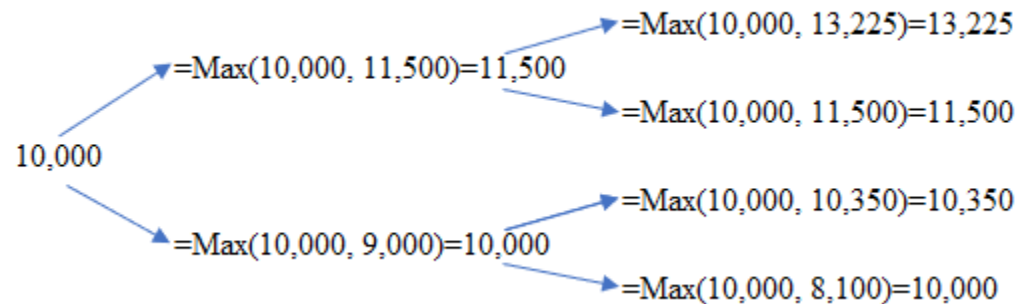
        Commentary on Question:
        *This part of the question was generally done poorly. While most candidates were
        able to calculate the account values at the end of year 2, fewer candidates were
        able to correctly calculate the GMAB benefit bases and resulting GMAB claims.
        Common errors included growing the GMAB guarantee with interest, assuming
        the GMAB benefit base was the same for the "up-down" and "down-up"
        scenarios, not recalculating the benefit base after each period, and incorrectly
        calculating the payoff as the difference between the account value and the
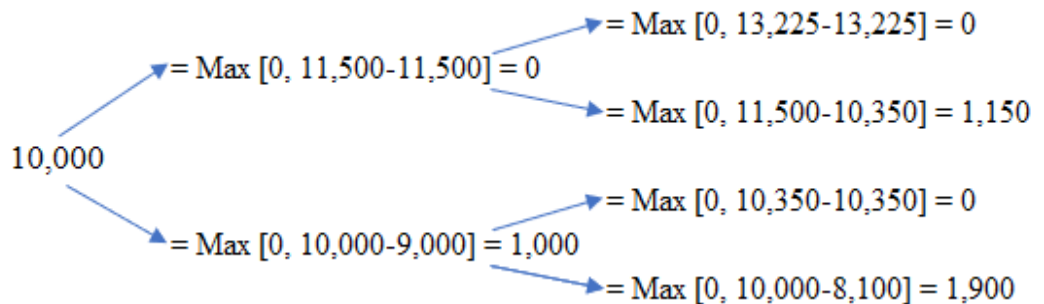   guarantee.*

4. **Continued**

Account Value:

$$10{,}000$$

$$=10{,}000*(1+15\%) = 11{,}500$$

$$=11{,}500*(1+15\%) = 13{,}225$$

$$=11{,}500*(1-10\%) = 10{,}350$$

$$=10{,}000*(1-10\%) = 9{,}000$$

$$=9{,}000*(1+15\%) = 10{,}350$$

$$=9{,}000*(1-10\%) = 8{,}100$$

GMAB benefit base:

$$10{,}000$$

$$=Max(10{,}000,\ 11{,}500)=11{,}500$$

$$=Max(10{,}000,\ 13{,}225)=13{,}225$$

$$=Max(10{,}000,\ 11{,}500)=11{,}500$$

$$=Max(10{,}000,\ 9{,}000)=10{,}000$$

$$=Max(10{,}000,\ 10{,}350)=10{,}350$$

$$=Max(10{,}000,\ 8{,}100)=10{,}000$$

GMAB Payoff = Max [0, GMAB Benefit Base - Account Value]

$$10{,}000$$

$$= Max\ [0,\ 11{,}500-11{,}500] = 0$$

$$= Max\ [0,\ 13{,}225-13{,}225] = 0$$

$$= Max\ [0,\ 11{,}500-10{,}350] = 1{,}150$$

$$= Max\ [0,\ 10{,}000-9{,}000] = 1{,}000$$

$$= Max\ [0,\ 10{,}350-10{,}350] = 0$$

$$= Max\ [0,\ 10{,}000-8{,}100] = 1{,}900$$

(b)   Calculate the risk neutral probability of the account value decreasing 10% in a given year.

Commentary on Question:
Most candidates did well on this part of the question. Some candidates used (1+Rf) instead of exp(Rf) and received partial credit.

4. **Continued**

$S(0) = 10,000; \ Su = S(0) * [1 + 15\%] = 11,500; \ Sd = S(0) * [1 - (10\%)] = 9,000$

$$\begin{cases} Pu + Pd = 1 \\ Pu * Su + Pd * Sd = S(0) * \exp(Rf) \end{cases} \qquad \begin{cases} Pu + Pd = 1 \\ Pu * 11,500 + Pd * 9,000 = 10,000 * \exp(0.06) \end{cases}$$

$Pu = 1 - Pd$

$11500 * (1 - Pd) + 9000 * pd = 10000 * \exp(0.06)$

$11500 - 11500 * Pd + 9000 * pd = 10000 * \exp(0.06)$

$(-11500 + 9000) * Pd = 10000 * \exp(0.06) - 11500$

$Pd = (10,000 * \exp(0.06) - 11,500) / (-2,500) = 0.3527$

(c)   Calculate the cost of hedging the GMAB rider at issue.

Commentary on Question:

Fewer candidates received full credit for this part of the question. Partial credit was given to candidates who had correct formulas but incorrect inputs (eg. payoffs from part a) of the question) as to not penalize candidates for the same mistake more than once. Common mistakes including not discounting the cost of hedging with interest, not including probabilities for two periods, and incorrectly including the cost of hedging for the first period.

Cost of hedge =

= $\exp(-Rf * T) * [\ Suu * Pu * Pu + Sud * Pu * Pd + Sdu * Pd * Pu + Sdd * Pd * Pd]$

= $\exp(-0.06 * 2) * [\ 0 * 0.6473 * 0.6473 + 1,150 * 0.6473 * 0.3527 + 0 * 0.3527 * 0.6473 + 1,900 * 0.3527 * 0.3527]$

= $\exp(-0.06 * 2) * [\ 0 + 1,150 * 0.6473 * 0.3527 + 0 + 1,900 * 0.3527 * 0.3527]$

= \$442.49

5.   **Learning Objectives:**
1.   The candidate will understand, evaluate and use stochastic, generalized linear, multi-state, projection and transition matrix models. The candidate will demonstrate an understanding of their underlying methodologies, strengths, limitations, and applications.

Learning Outcomes:
(1a)   With respect to stochastic models:
- Explain and apply the stochastic modeling methodology, including measurement metrics (e.g., CTE).
- Describe and apply the theory and uses of real world versus risk neutral assumptions.
- Describe and apply the techniques of Monte Carlo simulation (including variance reduction and importance sampling).
- Describe and evaluate Random Number Generator models, and explain their uses, advantages, and theory.
- Describe and evaluate how stochastic models may be used to understand mortality and policyholder behavior risks and inform the use of reinsurance.
- Describe the technique of nested stochastic projections and explain why they are needed, and evaluate implementation issues.

Sources:
LAM-135-19: Stochastic Modeling, Theory and Reality from and Actuarial Perspective, sections I.A, I.B-I.B.3.a, I.B.4 & I.D-I.D.3

Stochastic Modeling is on the Rise, Product Matters, Nov 2016

Commentary on Question:
Commentary listed underneath question component.

Solution:
(a)   Compare the use of True Random Number Generators and Pseudo Random Number Generators as a source of randomness in a stochastic model.

Commentary on Question:
The question asks candidates to describe and evaluate Random Number Generator models, and explain their uses, advantages, and theory. Most candidates performed well on this question. Some candidates only talked about Pseudo Random Number Generators without specifically describing True Random Number Generators, so were only given partial credit.

True Random Number Generators make use of naturally occurring events as the source of input for randomness and generate random numbers by detecting small and unpredictable changes in the real-world data

5. Continued

Pseudo Random Number Generators are mathematical algorithms that produce sequences of numbers that appear random. They are sequences which are deterministic and finite in nature

True Random Number Generators are a better source of randomness, but Pseudo Random Number Generators are preferred as they are much easier to model and much more efficient when operating in a stochastic environment

A key criteria in stochastic modeling is the random numbers should be reproducible to ensure changes to the stochastic model work correctly. This is a characteristic of Pseudo Random Number Generators but not True Random Number Generators.

(b)

    (i)   Identify how stochastic modeling could improve each deterministic mortality parameter.

    (ii)   Describe an approach which could be used to model each parameter stochastically.

Commentary on Question:
The question asks candidates to describe and evaluate how stochastic models may be used to understand mortality risks and why they are needed and how they can be implemented. Candidates generally did not perform well on this question. Many candidates only described why stochastic modeling is better than deterministic modelling in general without providing specific information on each mortality parameter.

Base Mortality Rate
A mortality study can be considered one random sample from the portfolio's "true" mortality. With any random sample, uncertainty exists as to whether the sample is a good representation of the population. The stochastic model can improve upon the deterministic model by modeling this uncertainty stochastically.

Model mortality as a binomial process where the experience study is the mean and 1/sqrt(#claims) is the standard deviation.

For a stochastic iteration, use the normal approximation to the binomial to randomly select a base mortality assumption for that iteration

Mortality Improvement Rate
Mortality Improvement rates vary significantly by attained age so a stochastic process could introduce variability and be expanded to individual ages as opposed to applying a single assumption to all ages

5. **Continued**

Using the mean and standard deviation of historical mortality improvements, a normal distribution could be used to generate stochastic mortality improvement scenarios

Catastrophic Mortality
The frequency and severity distributions for catastrophies can be modeled based on historical information. That additional mortality can be randomly sampled each year in the stochastic projection

(c)   Assess if the proposed capital amount is sufficient to cover expected losses at the 90th percentile. Justify your answer.

Commentary on Question:
The question asks candidates to explain and apply risk measurement metrics in a real-life scenario. It is expected that candidates would consider different metrics to give a full assessment on the scenario as different conclusion could be drawn depending on the chosen metric. Most candidates only based their assessment on one criteria e.g VaR. Another common mistake candidates made is to only include additional capital of $400 without adding average NPV of death benefit when determining the capital constraint. Candidates need to provide explanation to support the conclusion they are offering to receive full credit.

There are different ways to calculate VaR under the given scenario, all reasonable answer will be given full credit. The solution below only lists one of the possible answers.

Average NPV of death benefits = CTE(0) = $173.25M
Management proposes holding capital of $400M in addition to the average NPV of death benefits, so the total capital Management proposes to hold on balance sheet is $400 + $173.25= $573.25M

There are different risk metrics to measure expected losses at the 90th percentile.

- CTE(90) reflects the probability weighted loss at the 90th percentile. Given that there are 50 simulations, CTE(90) is calculated as the average of the worst 5 NPV of death benefit within those 50 simulation.
  CTE(90) = $599.23M > $573.25M

  As CTE(90) loss is greater than proposed capital amount, Senior Management is incorrect that they are sufficiently capitalized

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5. Continued

- VaR(90) reflects the loss at the 90th percentile.
  VaR(90) is the worst 46th NPV death benefit when sorting the 50 simulations.
  VaR(90) = $542.33M < $572.25M

  As VaR(90) loss is less than proposed capital amount, Senior Management is correct that they are sufficiently capitalized

6. **Learning Objectives:**
2. The candidate will understand and be able to assess issues and concerns common to actuarial models and their development and management.

Sources:
Model Validation for Insurance Enterprise Risk and Capital Models, 2014 (excluding Appendices)

ASOP 56: Modeling, sections 3 & 4

LAM-133-19: Model Risk Mitigation and Cost Reduction Through Effective Documentation

Commentary on Question:
This question tested candidates understanding of ASOP 56, model validation, and model documentation. In Part a candidates needed to critique, with justification, each statement to receive full credit, while in Part b just listing the types of documentation was sufficient for full credit. The majority of answers in part a were partially correct but needed more detail to receive full credit. In part b, most candidates received at least partial credit.

Solution:
(a) Critique each statement in the context of Model Risk Management and *ASOP 56, Modeling*.

A. *Your management team suggests to fully rely on MYB for modeling expertise as they control the functionality and are responsible for updating the calculation engines.*

B. *To assess usefulness of the report you decide to survey a group of actuaries who use the report for their feedback.*

C. *To mitigate third party risk your company has applied a margin of conservatism to the mortality table*

D. *MYB failed to deliver your stress testing model on time. As a result, your risk department is repurposing a pricing model to conduct their regulatory reporting*

Commentary on Question:
Candidates generally did well critiquing statements a and d of this question. For statement a, candidates generally did well highlighting the need for the company to review/understand the model—but did not mention the need to disclose reliance on MYB, resulting in partial credit. In part b, some candidates accurately identified the potential for bias and the need for additional individuals to be surveyed. A significant number of candidates did identify the problem with this approach and said the practice was appropriate, receiving no credit.

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6. Continued

Candidates struggled to earn full credit on part c, with few mentioning the inadvisability of adding conservatism at the input parameter level. Many candidates received partial credit for mentioning that mortality conservatism will not reduce third-party risk. In part d, most candidates highlighted the need to consider the intended use of the model and received full credit in doing so.

A.   This statement is incorrect. The company should review and understand the model to correctly use it and interpret the output. The modeling team should attempt to determine limitations of the model, which can be difficult due to vendors not wanting to disclose this information. Also, the extent to which the company relies on MYB should be disclosed.

B.   This statement is incorrect. Obtaining feedback from the users of the report will likely result in asymmetric feedback and bias. Users are likely to recommend the report if the report is favorable to their work and vice versa. Individuals who don't use the report directly should also be surveyed.

C.   This statement is incorrect. Adding conservatism to the mortality table will not address third-party risk. The company should instead review the model and mortality assumption to address third-party risk. Additionally, adding conservatism at the input parameter level is not advisable, as this contributes to opacity in the model.

D.   This statement is likely incorrect. It is important for the intended purpose of the model to match the use of the model. Pricing models are not intended to be used for regulatory reporting, as they usually include market consistent assumptions while regulatory reporting models contain more conservatism.

(b)   Identify additional documentation required from MYB and your company to ensure effective model risk control.

Commentary on Question:
Most candidates received at least partial credit on this section. Many did not name the three categories of documentation explicitly. Listing the types of documentation was sufficient for full credit, though some candidates also described the documentation. Full credit was given if detailed examples of the types of underlying documentation were described. Most candidates only highlighted one or two of the three areas, resulting in partial credit.

The user guide alone is not sufficient documentation to manage model risk. Comprehensive documentation includes model development documentation, model implementation documentation and on-going model governance documentation.

6. Continued

Examples of each include:

Model development documentation: User Guide, How model outputs will be used, company specific assumptions

Model implementation documentation: Development tests performed by MYB, Data Dictionary, Intended Use and Limitations

Model governance documentation: Model monitoring, model and assumption change management, implement changes, reviewing changes

7.   **Learning Objectives:**

4.   The candidate will understand the basic design and function of Economic Scenario Generators and Equity Linked Insurance Models.

Learning Outcomes:

(4b)   With respect to Equity-Linked models:
- Describe and apply methods for modeling long-term stock returns and certain guarantee liabilities (GMMB, GMDB, GMAB).
- Describe and evaluate the Actuarial and Hedging risk metrics for GMAB and GMDB models.
- Describe and apply methods for modeling Guaranteed annuity options and Guaranteed Minimum Income Benefits (GMIB), and EIA guarantees.

Sources:

Investment Guarantees, Hardy, Mary, 2003
- Ch. 1: Investment Guarantees
- Ch. 6: Modeling the Guarantee Liability
- Ch. 7: A Review of Option Pricing Theory (pp. 115-125)
- Ch. 8: Dynamic Hedging for Separate Account Guarantees (pp. 133-143)
- Ch. 13: Equity-Indexed Annuities

LAM-139-19: Simulation of a Guaranteed Minimum Annuity Benefit, Freedman, 2019; Excel Model - Stochastic Simulation of a GMAB Option (Accompanies Simulation of a GMAB)

Commentary on Question:
Commentary listed underneath question component.

Solution:

(a)   Calculate the cost of hedging the GMMB in the pricing model. Show all work.

Commentary on Question:
Candidates in general were able to project the GMMB account value and MER. Candidates are expected to include the profit estimate in the GMMB cost of hedging calculation in order to gain full credit.

GMMB Account Value RF

	0	1	2	3	4	5
BoP		100,000	98,700	97,417	96,150	94,901
Growth		5,000	4,935	4,871	4,808	4,745
MER		-6,300	-6,218	-6,137	-6,057	-5,979
Withdrawal						-93,667
EoP	100,000	98,700	97,417	96,150	94,901	0

7. **Continued**

MER = 30,692
Claims = -6,333
Gross Profit = 24,358
Net Profit = 10,00
The cost of hedging the GMMB in the pricing model is 14,358

(b) Calculate the profit or loss generated by the EIA

Commentary on Question:
The first part of the question asked for the calculation of projected account value and guarantee value of the EIA. Candidates in general were able to gain some credit by correctly identifying the participation rate, floor and cap and calculating AV and GV.

Most candidates had challenges with the second part of the question, especially in connecting the part(a) to the put-call parity and demonstrating their understanding of how to calculate the profit or loss generated by the EIA.

EIA
RF

Period	Return	Participation	Floor	Cap	Actual	AV	GV	Max
0						100,000	100,000	
1	5.0%	4.5%	2.0%	10.0%	4.5%	104,500	104,000	104,500
2	5.0%	4.5%	2.0%	10.0%	4.5%	109,203	108,160	109,203
3	5.0%	4.5%	2.0%	10.0%	4.5%	114,117	112,486	114,117
4	5.0%	4.5%	2.0%	10.0%	4.5%	119,252	116,986	119,252
5	-50.0%	-45.0%	2.0%	10.0%	2.0%	121,637	121,665	121,665

To calculate the call cost by the put-call parity

$$p_t + S_t = c_t + Ke^{-r(T-t)}$$

P = 14,358
S = 100,000
Ke(-rt) = 100,000 e(-0.03 x 5)
C = 28,288

The remaining funds to purchase a zero-coupon bond = 100,000 – 28,888 = 71,712
Value of zero-coupon bond at time 5 = 71,712 *(1+4%)^5 = 87,249

Call Payout = 121,637 – 100,000 = 21,637
EIA Claim = -121,655

Profit = 87,249 + 21,637 – 121,655 = -12,779

8. **Learning Objectives:**
1.   The candidate will understand, evaluate and use stochastic, generalized linear, multi-state, projection and transition matrix models. The candidate will demonstrate an understanding of their underlying methodologies, strengths, limitations, and applications.

Learning Outcomes:
(1b)   With respect to generalized linear models:
   • Describe and apply the basic principles of GLMs, and evaluate where GLMs might be useful in a Life Insurance context.

Sources:
LAM-138-19: A Practitioner's Guide to Generalized Linear Models, 1.1-1.108, 1.118-1.130 & 3.1-3.14

Commentary on Question:
Commentary listed underneath question component.

Solution:
(a)   Compare each of the following components when implementing a linear model versus a generalized linear model:

   • Random Component
   • Systematic Component
   • Link Function

Commentary on Question:
Candidates generally did well on explaining the differences of Random and Systematic Components, however most candidates failed to point out that the link function of GLM should be monotonic and differentiable. A small portion of the candidates mixed up the different components which did not receive credit.

Random Component
For a linear model the random component is assumed to be independent and normally distributed. The means for each component can be different but they all have the same variance. These conditions for LM may be hard to assert when modeling so the GLM relaxes these assumptions.
GLM assumes that the random component is independent and from one of the exponential family of distributions and the variance is permitted to vary with the mean of the distribution.

Systematic Component
In both LM and GLM, the p covariates are combined to give the linear predictor n
n = X.beta

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8. **Continued**

Link Function
The link function is the relationship between the random and systematic components.
For a LM the link function is equal to the identity function. $E[Y] = mu = n$.
For GLM the link function g is differentiable and monotonic. $E[Y] = mu = g^{-1}(n)$. This allows for transformation of Y such as $\ln[Y]$.

(b)   Calculate the expected claim frequency for a 35-year-old male smoker. Show all work.

Commentary on Question:
Candidates generally did well on the linear part but failed to correctly use the link function.

$Y = \exp(3.2+0.02*\text{Issue age} + 0.1*(\text{Gender: 1 for male, 0 for female}) - 0.4*(\text{Smoker: 1 for non-smoker, 0 for smoker})$

$Y = \exp(3.2+0.02*35+0.1*1-0.4*0) = 54.598$

(c)
(i)   Describe challenges a company may face when implementing a GLM

(ii)   Describes techniques which could be used to overcome these challenges

Commentary on Question:
Candidates generally did well on Part (c).

(i) Challenges:
In Insurance modeling there are large datasets to be modeled which makes it not practical to find values of beta maximizing likelihood using explicit techniques and linear algebra.

In real life there are many factors each with many levels to consider when implementing a GLM.

Challenges gathering and cleaning data such that it can be appropriately used in development and testing of GLM.

Upfront learning curve to understand GLMs and their uses, misunderstanding/misinterpreting of output resulting in incorrect decision making.

8. **Continued**

(ii) Solutions:
Can use iterative methods such as the Newton-Raphson iteration or other generic commercial packages such as SAS, R, S+ which fit GLMs more quickly.

It is helpful to parameterize the GLM with an intercept term which is a parameter that applies to all observations.

Develop a data warehouse to consolidate, maintain and ensure consistent formatting of data.

Conduct internal training for employees on GLMs and interpretation of model output. Consult with subject experts on GLMs to assist with implementation / understanding of output.