****BEGINNING OF EXAMINATION****

1. Which of these statements is **<u>not</u>** true with respect to expenses incurred by the insurer?

(A) Commissions are often paid to an agent in the form of a high percentage of the first year's premium plus a lower percentage of subsequent premiums.

(B) In calculating gross premiums, specific allowances are sometimes made for termination expenses.

(C) Initial expenses cover the costs to produce annual statements to policyholders.

(D) Per-policy renewal costs are often assumed to increase to account for inflation over the term of the policy.

(E) Underwriting expenses may vary according to the amount of the death benefit.

2. The one-hour mortality rate for a population of 100,000 newly hatched insects is 0.6.

Two models are used for the fractional distribution of deaths between integer hours. Model (U) assumes deaths are uniformly distributed between integer hours. Model (C) assumes a constant force of mortality between integer hours.

Let d^u denote the expected number of deaths between 20 minutes and 50 minutes after hatching under Model (U) and let d^c denote the expected number of deaths between 20 minutes and 50 minutes after hatching under Model (C), for these 100,000 insects.

Calculate $d^u - d^c$.

(A) 2884

(B) 2901

(C) 2919

(D) 2948

(E) 2967

3. At time 0 a mortality study has 50 participants. At time 1.1, ten of them die. At time 1.5, N additional participants enter the study. At time 1.9, six more of them die. There are no other entries, exits, or deaths.

The Kaplan-Meier estimate of survival to time 3 is 72%.

Calculate N.

(A) 12

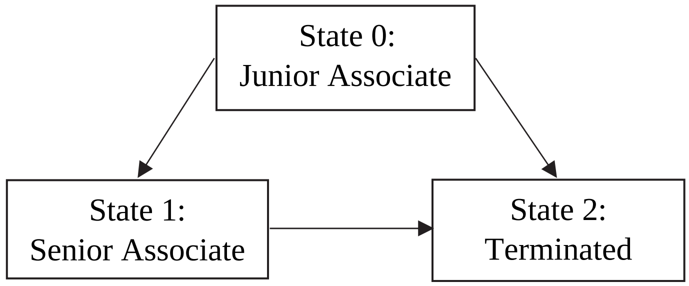
(B) 14

(C) 16

(D) 18

(E) 20

4. Employees at NED company have the following model of employment:



Each year the Junior Associates have an 80% probability of remaining in State 0, a 15% probability of ending the year as a Senior Associate, and a 5% probability of being terminated during the year.

Each year the Senior Associates have a 90% probability of remaining in State 1, and a 10% probability of being terminated during the year.

Zhanyi is currently a Junior Associate, and Yuanyuan is currently a Senior Associate. Their transitions are independent.

Calculate the probability that both individuals are still employed by NED after one year but are both Terminated by the end of the second year.

(A) 0.005

(B) 0.010

(C) 0.015

(D) 0.020

(E) 0.025

5. You are given the following information about a double decrement model.

Age, x	$l_x^{(\tau)}$	$q_x^{(1)}$	$q_x^{(2)}$
84	10,000	---	0.030
85	9,250	$q_{84}^{(1)} + 0.015$	---
86	8,278	0.070	0.060

Calculate $d_{85}^{(2)}$.

(A) 407

(B) 417

(C) 427

(D) 437

(E) 447

6. A life insurer issues a 10-year term insurance to (55), with sum insured 100,000 payable at the end of the year of death.

You are given:

(i) Mortality follows the Standard Ultimate Life Table.

(ii) $i = 0.05$

Calculate the 98th percentile of the present value of the benefit.

(A) 61,391

(B) 64,461

(C) 67,684

(D) 71,068

(E) 74,621

7. For a 2-year deferred, 2-year term insurance of 10,000, issued to a newly selected life age 75, you are given:

(i) Benefits are payable at the end of the year of death.

(ii) $i = 0.05$

(iii) The following select and ultimate mortality table:

Age, x	$q_{[x]}$	$q_{[x]+1}$	$q_{[x]+2}$	q_{x+3}	$x+3$
75	0.08	0.10	0.15	0.20	78
76	0.09	0.12	0.18	0.22	79
77	0.10	0.15	0.20	0.25	80
78	0.12	0.18	0.22	0.27	81
79	0.15	0.20	0.25	0.30	82

Calculate the expected present value of the insurance.

(A) 2120

(B) 2230

(C) 2440

(D) 2520

(E) 2610

8. You are given the following information about a communications satellite:

(i) The satellite has two components: Component I and Component II.

(ii) The satellite functions until either component fails.

(iii) The loss of satellite function causes an immediate loss of 400.

(iv) If the cause of the loss was the failure of Component II, an additional loss of 300 also occurs immediately.

(v) The force of failure for Component I is $\mu_{x+t}^{(I)} = 0.01$, $t \geq 0$

(vi) The force of failure for Component II is $\mu_{x+t}^{(II)} = 0.02$, $t \geq 0$

(vii) $\delta = 0.05$

Calculate the expected present value of the future loss.

(A) 205

(B) 210

(C) 215

(D) 220

(E) 225

9. You are given the following information:

(i) $i = 0.05$

(ii) For a fully discrete whole life insurance of 1000 on (x), the annual net premium is 11.120.

(iii) For an n-year fully discrete endowment insurance of 1000 on (x), the annual net premium is 76.529.

Calculate the annual net premium for a whole life insurance of 1000 on (x), with premiums payable for the first n years.

(A) 16.6

(B) 20.1

(C) 23.5

(D) 27.3

(E) 30.5

10. A life insurer issues a fully discrete whole life insurance with a face amount of 100,000 to (45).

You are given:

(i) Annual premiums are 1,045.

(ii) Mortality follows the Standard Ultimate Life Table.

(iii) Initial expenses are 100 plus 50% of the first year's premium.

(iv) Renewal expenses are 5% of the renewal premiums.

(v) Claim expenses of 500 are incurred when the death benefit is paid.

(vi) $i = 0.05$

Calculate the standard deviation of the gross future loss at issue for this policy.

(A) 12,902

(B) 12,956

(C) 13,051

(D) 13,095

(E) 14,154

11. A life insurer issues a whole life insurance to (45) with a face amount of 100,000.

You are given:

(i) Annual premiums are 5,280 and are paid continuously.

(ii) The sum insured is paid immediately on death.

(iii) Initial expenses are 100.

(iv) Renewal expenses are 10% of the renewal premiums and are paid continuously.

(v) $i = 0.05$

Calculate the minimum value for the future lifetime of (45), such that the insurer makes a profit on this policy.

(A) 13.5

(B) 14.5

(C) 15.5

(D) 16.5

(E) 17.5

12. A sickness insurance policy is issued to a Healthy life age 50, with a 10-year term.

The benefit is a payment of 5000 every ½-year if the policyholder is Sick on the first day of the ½-year.

Level premiums are payable ½-yearly, conditional on the policyholder being Healthy on the premium date.

You are given:

(i) Sickness and mortality follow the Standard Sickness-Death model.

(ii) Annuity values are calculated using Woolhouse's 2-term formula.

(iii) $i = 0.05$

(iv) $\ddot{a}^{(2)00}_{50:\overline{10}|} = 7.5385$

Calculate the ½-yearly premium.

(A) 124

(B) 127

(C) 130

(D) 133

(E) 136

13. A couple, who are both aged 50, purchase a deferred annuity, with a 10-year deferred period.

Level premiums are payable annually during the deferred period, conditional on both lives surviving.

The policy benefits are as follows:

- If either life dies during the deferred period, a death benefit of 400,000 is paid at the end of the year of death, and there are no further benefits.
- If both lives survive the deferred period, an annuity of 60,000 per year is paid annually in advance until the second death.

You are given the following information:

(i) The lives have independent future lifetimes.

(ii) Mortality of each life follows the Standard Ultimate Life Table.

(iii) $i = 0.05$

(iv) The term joint life insurance function, $A^{\;\;1}_{\overline{50:50}:\overline{10|}} = 0.02896$.
Note: The figure above the 50:50 in item (iv) is a bracket (┌─┐), not a horizontal line (—).

Calculate the annual net premium.

(A) 73,580

(B) 74,690

(C) 75,790

(D) 76,600

(E) 77,710

14. For a fully discrete whole life insurance of 1000 on (50), you are given:

(i) First year expenses are 30% of the gross premium plus 300.

(ii) Renewal expenses are 4% of the gross premium plus 30.

(iii) All expenses are incurred at the beginning of the policy year.

(iv) Gross premiums are calculated using the equivalence principle.

(v) Mortality follows the Standard Ultimate Life Table.

(vi) $i = 0.05$

Calculate the gross premium reserve at the end of the first policy year.

(A) -181

(B) -212

(C) -241

(D) -272

(E) -301

15. You are given:

(i) $q_{36} = 0.00041$

(ii) $i = 0.03$

(iii) $\ddot{a}_{37:\overline{23}|} = 16.8078$

Calculate the Full Preliminary Term reserve at time 2 for a 25-year fully discrete endowment insurance, issued to (35), with sum insured 100,000.

(A) 2900

(B) 2910

(C) 2940

(D) 2950

(E) 2970

16. Steve and Jeff are both age 50 and have independent future lifetimes. They purchase an insurance policy which provides 1,000,000 payable at the end of the year of Steve's death, provided Steve dies before Jeff.

Annual premiums are payable while Steve and Jeff both survive.

You are given:

(i) Mortality follows the Standard Ultimate Life Table.

(ii) $i = 0.05$

(iii) The annual net premium is 7,797.

Calculate the net premium reserve at time 20, assuming both lives are alive at that time.

(A) 184,646

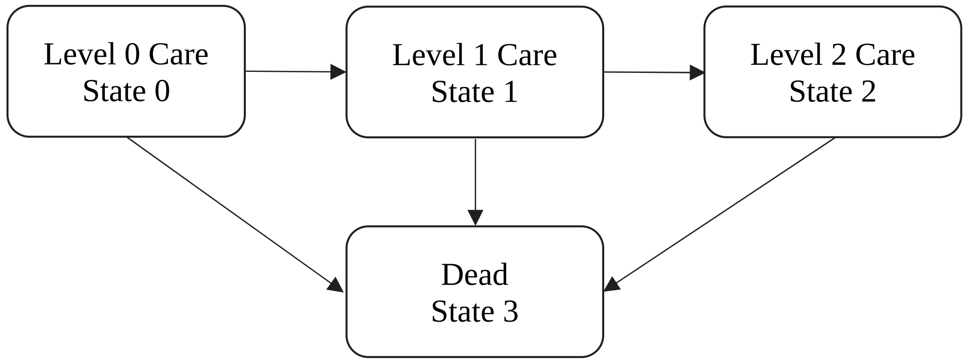
(B) 194,757

(C) 205,882

(D) 216,842

(E) 223,698

17. A CCRC uses the following multiple state model:



You are given:

(i) The monthly costs, per resident, incurred to cover maintenance of the facility, medical care and services, and other related expenses, depend on the level of care as follows:

- Level 0 Care: 3,500
- Level 1 Care: 8,000
- Level 2 Care: 20,000

Costs are assumed to be incurred at the start of each month.

(ii) Costs of 50,000 are incurred on the death of the resident.

(iii) Fees in each State are 4000 per month, at the start of each month.

(iv) There are no other costs or expenses.

(v) $i = 0.05$

(vi) $\ddot{a}_{75}^{(12)00} = 8.751$; $\ddot{a}_{75}^{(12)01} = 0.754$; $\ddot{a}_{75}^{(12)02} = 0.397$; $\bar{A}_{75}^{03} = 0.178$

(vii) The CCRC holds reserves equal to the expected present value of future costs minus the expected present value of future fees.

Calculate the reserve required for a resident age 75, in Level 0 Care.

(A) 64,450

(B) 65,540

(C) 66,630

(D) 67,720

(E) 68,810

18. HLP insurance sells a 5-year term insurance contract to (50).

You are given:

(i) The risk discount rate is 12%

(ii) Mortality follows the Standard Ultimate Life Table.

(iii) There are no lapses.

(iv) The profit signature is $\Pi = (-350, 125, 130, 135, 140, 145)$

(v) $\ddot{a}_{50:\overline{5}|} = 4.0278$ at $i = 0.12$

(vi) The profit margin is 1.45%

Calculate the premium.

(A) 1770

(B) 2020

(C) 2270

(D) 2520

(E) 2770

19. For a defined benefit pension plan, you are given:

(i) An employee is currently age 45 with 12 years of service.

(ii) The salaries in the past two years were 79,000 and 81,000.

(iii) The retirement benefit is a monthly annuity due payable from age 65 based on the two-year final average salary.

(iv) The accrual rate is 2.0% per year of service.

(v) There are no exits from the plan other than death and retirement.

(vi) No benefits are payable upon death before retirement.

(vii) Mortality follows the Standard Ultimate Life Table.

(viii) $i = 0.04$

(ix) At $i = 0.04$, $\ddot{a}_{65}^{(12)} = 14.3714$.

Calculate the accrued liability for the employee's retirement benefit under the traditional unit credit method.

(A) 117,425

(B) 118,156

(C) 119,252

(D) 120,267

(E) 121,775

20. You are given the following for a retired life currently age 70:

(i) The current health care premium at age 70 is 3,000.

(ii) The premium is assumed to increase each year with a rate of health inflation of $j = 0.04$

(iii) The premium is also assumed to increase for each year of age by a factor of $c = 1.03093$

(iv) $i = 0.04$

You are also given the following table:

i^*	$\ddot{a}_{70}$ at $i = i^*$	i^*	$\ddot{a}_{70}$ at $i = i^*$
-4%	30.9953	1%	17.1346
-3%	27.1070	2%	15.5430
-2%	23.9013	3%	14.1843
-1%	21.2387	4%	13.0170
0%	19.0112	5%	12.0083

Calculate the expected present value of the post-retirement health benefits for this life.

(A) 36,025

(B) 46,629

(C) 63,716

(D) 71,704

(E) 81,321

1.

(*10 points*) NED Life issues a fully discrete whole life insurance of 100,000 on *(50)* with a return of premium benefit subject to the following conditions:

- If death occurs in the first 10 years, no premiums are returned.
- If death occurs after 10 years, all premiums, including the first 10, are returned without interest at the end of the year of death.

You are given:

(i) The gross premium is 2,000.

(ii) Premium expenses, payable at the beginning of the year, are 80% of the first year's premium and 5% of premium in subsequent years.

(iii) Termination expenses of 1,000 are paid at the end of the year of death.

(iv) Mortality follows the Standard Ultimate Life Table.

(v) $i = 0.05$

(vi) L^g is the gross loss at issue random variable.

(vii) $(IA)_{60} = 6.63303$

(viii) ${}_kV^g$ denotes the gross premium reserve at the end of year k.

(a) (*1 point*) Show that if $T_{50} = 11.8$, then L^g is 53,420 to the nearest 10. You should calculate the value to the nearest 1.

(b) (*2 points*) Calculate $\mathrm{E}[L^g]$.

(c) (*2 points*) Show that ${}_{10}V^g$ is 20,070 to the nearest 10. You should calculate the value to the nearest 1.

1. **Continued**

NED Life had 1000 such policies in force at the end of 15 years. You are given:

(i) ${}_{15}V^g = 34{,}333.78$ and ${}_{16}V^g = 37{,}480.51$

(ii) During year 16:

- There were 7 deaths.
- i = 0.052
- Premium expenses were 4% of premium.
- Termination expenses were 2,000 per death.

(d) (*4 points*)

(i) Calculate the total gain in the 16th year.

(ii) Calculate the gain by source in the following order: expenses (E), interest (I) and mortality (M).

(e) (*1 point*) An actuary at NED Life was asked to calculate the gain by source. He wasn't sure which order to use, so he calculated the gain based on all six possible orders. Referring to the sources of gain in order by their first letters, the order you did in part (d) was EIM.

Without further calculation, state, using the three-letter ids, which, if any, of the other 5 orders had the same gain from interest as EIM. Justify your response.

2.

(*10 points*) XYZ Insurance Company sells two year term policies to lives age *x*. The death benefit of 1,000,000 is payable at the end of the year of death. Premiums are payable annually.

The insurer holds reserves of 0 for this policy at time $t=0$ and at time $t=1$.

The assumptions used for profit testing this policy are:

(i) Precontract underwriting expenses are 3,000 per policy.

(ii) Commissions are 20% of premium in year 1 and 5% of premiums each year thereafter. The extra 15% of premium commissions in the first year is treated as a pre-contract cash flow for profit testing purposes.

(iii) Maintenance expense is 100 per policy at the beginning of every year including the first year.

(iv) The earned interest rate is 7%.

(v) Mortality is $q_{x+t} = 0.05 + 0.05t$ for $t = 0,1,2$

(vi) At the end of the first year 10% of the surviving policyholders withdraw.

(vii) There are no cash values.

(viii) $\Pr_1 = 31{,}213$.

XYZ uses a gross premium of 80,000 in its first profit test for this two year term policy.

(a) (*1 point*) Determine $\Pr_0$.

(b) (*2 points*) Determine $\Pr_2$.

(c) (*1 point*) Determine the profit signature for this policy.

(d) (*1 point*) The NPV for this policy using a risk discount rate of 20% is negative. Calculate the NPV to the nearest 1.

(e) (*1 point*) Calculate the internal rate of return for this policy.

2. **Continued**

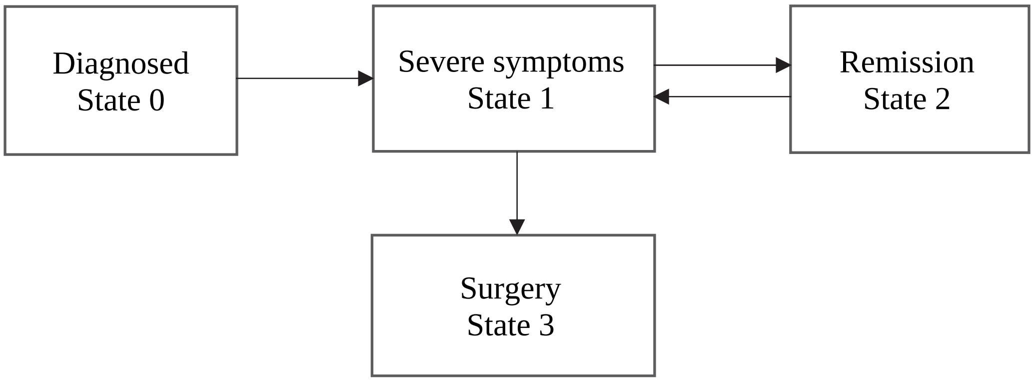
(f) (*1 point*) Your boss questions how you can have a positive IRR but a negative NPV. Explain how this is possible.

(g) (*2 points*) Calculate the NPV if the lapse rate increases to 15%.

(h) (*1 point*) Policies where profits increase as the lapse rate increases are called lapse supported policies. Explain why such policies are risky to the insurance company.

3.

(*9 points*) You are analyzing costs associated with a medical condition. You model the progression of the condition after a new diagnosis using the following 4-state model.



You are given the following information. Time t is measured in years from the date of diagnosis.

(i) $\mu_t^{01} = a + be^{ct}$ where $a = 0.4, b = 2.0,$ and $c = 0.7$;

(ii) $\mu_t^{12} = 0.2$; $\mu_t^{21} = 0.4$; $\mu_t^{13} = 0.1$

(iii) $\delta = 0.04$

(iv) $\bar{a}_0^{01} = 2.930$; $\bar{A}_0^{01} = 1.5185$

(a) (*3 points*)

(i) Show that, in this case, $\dfrac{\bar{A}_0^{03}}{\bar{a}_0^{01}} = \mu_0^{13}$.

(ii) Calculate the value of $\bar{a}_t^{\overline{11}}$.

(iii) State with reasons whether $\bar{a}_t^{\overline{11}}$ is bigger, smaller or the same as $\bar{a}_t^{11}$.

3. **Continued**

(b) (*4 points*) The cost of nursing care in State 1 is 12,000 per year, incurred continuously. The cost of surgery is 22,000. There are no nursing care costs in State 0 or State 2.

(i) Calculate the expected present value of the cost of surgery for a newly diagnosed patient.

(ii) Calculate the expected present value of the nursing care costs for a newly diagnosed patient.

(iii) COVID-19 precautions result in additional costs at a rate of 8,000 per year for the first 4 months of each period in State 1.

Write down an integral expression for the additional expected present value, using continuous sojourn annuities.

(c) (*2 points*) As a result of reorganization, the value of μ_t^{13} is increased to 0.2. All other transition intensities stay the same.

(i) State with reasons whether the expected present value of the cost of nursing care will increase, decrease, or stay the same.

(ii) State with reasons whether the expected present value of the cost of surgery will increase, decrease, or stay the same.

4.
(*10 points*)

(a) (*1 point*) Show that $e_x = e_{x:\overline{n}|} + {}_n p_x \; e_{x+n}$.

(b) (*2 points*) You are given:

(i) Mortality follows the Standard Ultimate Life Table (SULT).

(ii) $e_{87} = 6.56$

Show that $e_{90} = 5.2$ to the nearest 0.1. Calculate the value to the nearest 0.01.

(c) (*3 points*) Let $H = \min(3, K_{87})$ denote the 3-year temporary curtate future lifetime of (87). Calculate the standard deviation of H.

4. **Continued**

A mortality improvement scale is used to model mortality trends over the next three years for lives under the age of 90. There will be no mortality improvement for lives aged 90 or older.

You are given that the mortality rate for age x in year t to t+1 is,

$$q(x,t) = q(x,0)(1-\varphi(x))^t \quad t = 0,1,2,3$$

where

$$\varphi(x) = \begin{cases} 0.01 + \left(\dfrac{100-x}{1000}\right) & x < 90 \\ 0 & x \geq 90 \end{cases}$$

You are also given that

(i) The base mortality, $q(x,0)$ follows the SULT.

(ii) The notation $\tilde{e}_x$ is used for the expected value of the curtate future lifetime of a life aged *x* at time 0.

(d) (*2 points*)

(i) Calculate the three-year curtate life expectancy of a life age 87 at time 0, $\tilde{e}_{87:\overline{3}|}$.

(ii) Calculate $\tilde{e}_{87}$.

(e) (*2 points*)

(i) Sketch a graph illustrating the phenomenon that is commonly referred to as the "rectangularization of mortality."

(ii) State, with justification, whether a mortality improvement scale such as the one defined above will cause a rectangularization of mortality.

5.

(*7 points*) ABC Insurance issues a 10-year endowment insurance policy to Alex who is 60 and Chris who is 50. The sum insured is 200,000 payable at the earliest of the end of the year of the first death and the end of 10 years. Premiums are paid annually in advance.

You are given:

(i) Mortality follows the Standard Ultimate Life Table.

(ii) $i = 0.05$

(iii) Alex and Chris have independent future lifetimes.

(iv) Gross premiums are calculated using the equivalence principle.

(a) (*1 point*) Show that the net premium is 15,780 to the nearest 10. You should calculate the value to the nearest 1.

(b) (*2 points*) Show that ${}^2A_{50:60:\overline{10|}} = 0.39$ to the nearest 0.01. You should calculate the value to the nearest 0.0001.

(c) (*1 point*) Calculate the standard deviation of the net Loss at Issue random variable.

(d) (*1 point*) Expenses are 25% of the first year's premium plus 10% of all renewal premiums. Calculate the gross premium using the equivalence principle.

(e) (*2 points*) Following a major economic upheaval, the interest rate has increased to 10.25% per year. Note that $1.05^2 = 1.1025$.

(i) Calculate the percentage reduction in the gross premium, as a result of the change in the interest rate.

(ii) State with reasons whether the percentage reduction in the premium for a 10-year term joint life insurance on Alex and Chris would be greater or less than the change in (i) above.

6.

(*10 points*) Jane, who is 57, is a member of a final average salary defined benefit pension plan. She has 35 years service. You are given the following information. Jane's salary was 100,000 last year and this year.

- The accrual rate is 1.8%
- The pension is payable monthly to the plan member from age 65, with a 100% survivor pension paid to the member's partner after the member's death, provided this occurs after age 65.
- There is no benefit on death after withdrawal before age 65.
- The final average salary is the average of the final 2-year's salary before exit.
- Salaries are expected to be frozen for the next year.
- Jane's partner, Mike, is also 57.

You are also given the following valuation assumptions and information:

(i) Decrements follow the Standard Service Table

(ii) $i = 0.05$

(iii) Mortality after exit follows the Standard Ultimate Life Table.

(iv) Withdrawals occur ½ way through the year.

(v) Jane and Mike have independent future lifetimes.

(vi) $\ddot{a}_{65}^{(12)} = 13.0870$; $\ddot{a}_{65:65}^{(12)} = 11.2158$

(vii) Uniform distribution of deaths between integer ages for all other fractional age calculations.

(viii) a_x^w represents the value on withdrawal at age x of an annuity of 1 per year paid monthly in advance from age 65, including the survivor's benefit.

(ix) $a_{58.5}^w = 10.5804$; $a_{59.5}^w = 11.1456$

(x) The valuation uses the traditional unit credit funding method.

6. **Continued**

(a) (*2 points*)

(i) Show that, under the Standard Ultimate Life Table, ${}_{7.5}E_{57.5:57.5} = 0.655$ to the nearest 0.001. You should calculate the value to the nearest 0.0001.

(ii) Show that $a^{w}_{57.5} = 10.05$ to the nearest 0.01. You should calculate the value to the nearest 0.001.

(b) (*3 points*) Show that the actuarial liability for the withdrawal benefit is 36,000 to the nearest 100. You should calculate the value to the nearest 1.

(c) (*2 points*) Calculate the normal contribution for the withdrawal benefit.

(d) (*2 points*) Jane withdraws at age 57.5 and immediately gets divorced. Under the divorce settlement, Mike will receive a pension of $X/3$ for life from age 65, and Jane will receive a pension of X for life from age 65. The value of the settlement is equal to the value of the joint and last survivor pension payable had the couple not divorced.

Calculate X.

(e) (*1 point*) For each of the following cases, state whether the total benefit paid in the year from age 65 to 66 is greater before the divorce settlement or after.

(i) Jane dies before age 65, and Mike survives to age 66.

(ii) Mike dies before age 65, and Jane survives to age 66.

****END OF EXAMINATION****